MAT137 - Term 2, week 3, lecture 1

- **Reminder:** Problem Set 7 is due on Thursday, 30 January.
- Today's lecture will assume you have watched up to and including video 8.4.

  For Thursday's lecture, please watch videos 8.5 through 8.7 (the remaining videos on playlist 8).

Defining functions with integrals

Let f be an integrable function defined on an interval $[a, b]$.
Let c and d be elements of $[a, b]$.

Problem 1. (We did this last class.) Consider the following expressions.

1. $\int_c^x f(x)\,dx$
2. $\int_c^x f(u)\,du$
3. $\int_c^x f(c)\,dt$
4. $\int_c^x f(t)\,dt$

Which of these expressions make sense? Do any of them equal one another?

Problem 2. Define two functions F_c and F_d on $[a, b]$ by:

$$F_c(x) = \int_c^x f(t)\,dt \quad \text{and} \quad F_d(x) = \int_d^x f(t)\,dt.$$

What is the relationship between the values of F_c and F_d?

The first FTC

The following statement of this theorem is like the one in the Playlist 8 practice problems.

Theorem (First Fundamental Theorem of Calculus)

Suppose f is integrable on $[a, b]$, and let $c \in [a, b]$. Then the function

$$F(x) = \int_c^x f(t)\,dt$$

is continuous on $[a, b]$.

F is differentiable at any point x where f is continuous.
If x is such a point, then $F'(x) = f(x)$.

The video only states the theorem for a continuous function f.
In that case, F is simply an antiderivative for f on $[a, b]$.

True or false?

Let f and g be differentiable functions with domain $\mathbb{R}$.

Assume that $f'(x) = g(x)$ for all x.

Which of the following statements must be true?

1. $f(x) = \int_0^x g(t)\,dt$.
2. If $f(0) = 0$, then $f(x) = \int_0^x g(t)\,dt$.
3. If $g(0) = 0$, then $f(x) = \int_0^x g(t)\,dt$.
4. There exists a $C \in \mathbb{R}$ such that $f(x) = C + \int_0^x g(t)\,dt$.
5. There exists a $C \in \mathbb{R}$ such that $f(x) = C + \int_1^x g(t)\,dt$.

True, false, or can't decide?

We want to find a function H with domain $\mathbb{R}$ such that $H(1) = -2$ and such that

$$H'(x) = e^{\sin x} \quad \text{for all } x.$$

Decide whether each of the following statements is true, false, or you do not have enough information to decide.

1. The function $H(x) = \int_0^x e^{\sin t}\, dt$ is a solution.
2. The function $H(x) = \int_2^x e^{\sin t}\, dt$ is a solution.
3. $\forall C \in \mathbb{R}$, the function $H(x) = \int_0^x e^{\sin t}\, dt + C$ is a solution.
4. $\exists C \in \mathbb{R}$ s.t. the function $H(x) = \int_0^x e^{\sin t}\, dt + C$ is a solution.
5. The function $H(x) = \int_1^x e^{\sin t}\, dt - 2$ is a solution.
6. There is more than one solution.