MAT137 - Term 2, week 3, lecture 2

- **Reminder:** Problem Set 7 is due on Thursday, 30 January.
- Today's lecture will assume you have watched up to and including video 8.7.

  For tomorrow's lecture, please watch videos 9.1 through 9.4.

True, false, or can't decide?

We want to find a function H with domain $\mathbb{R}$ such that $H(1) = -2$ and such that

$$H'(x) = e^{\sin x} \quad \text{for all } x.$$

Decide whether each of the following statements is true, false, or you do not have enough information to decide.

1. The function $H(x) = \int_0^x e^{\sin t}\, dt$ is a solution.
2. The function $H(x) = \int_2^x e^{\sin t}\, dt$ is a solution.
3. $\forall C \in \mathbb{R}$, the function $H(x) = \int_0^x e^{\sin t}\, dt + C$ is a solution.
4. $\exists C \in \mathbb{R}$ s.t. the function $H(x) = \int_0^x e^{\sin t}\, dt + C$ is a solution.
5. The function $H(x) = \int_1^x e^{\sin t}\, dt - 2$ is a solution.
6. There is more than one solution.

Using FTC1

Our primary use for FTC1 in this course is to prove FTC2. But it does have some applications of its own as a new differentiation rule.

Problem 1. Find the derivative of $F(x) = \int_0^x \cos^2 t \, dt$.

Problem 2. Find the derivative of $G(x) = \int_x^7 7te^{t^2} \, dt$.

(Trickier) Problem 3. Find the derivative of $H(x) = \int_0^{x^2} \frac{\sin t}{1+t} \, dt$.

Hint. The answer is not $\frac{\sin(x^2)}{1+x^2}$.

Hint 2. Try to express $H(x)$ in terms of $F(x) = \int_0^x \frac{\sin t}{1+t} \, dt$.

Going further...

(Trickier) Problem 4. Find the derivative of $S(x) = \int_0^x xe^{t^2}\,dt$.

Problem 5. Find the derivative of $F(x) = \int_{x^2}^{\sin x} \arctan t\,dt$.

Hint: Try to write $F(x)$ as a sum of two functions whose derivatives you know how to compute.

Okay! We're [trying to be] mathematicians, so it's time to generalize! Why solve one example, when we can solve **all** the examples?

Exercise

Let f, u, v be differentiable functions on $\mathbb{R}$, and define:

$$F(x) = \int_{u(x)}^{v(x)} f(t)\,dt.$$

Find a formula for $F'(x)$ in terms of some or all of f, u, v, f', u', v'.

FTC2: Definite integrals

We didn't spend time on these in class, but I'm posting them as extra practice problems.

Compute the following definite integrals:

1. $\displaystyle\int_1^2 x^3\,dx$
2. $\displaystyle\int_0^1 \left[e^x + e^{-x} - \cos(2x)\right]\,dx$
3. $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\,dx$
4. $\displaystyle\int_{\pi/4}^{\pi/3} \sec^2 x\,dx$
5. $\displaystyle\int_1^2 \left[\frac{d}{dx}\left(\frac{\sin^2 x}{1+\arctan^2 x + e^{-x^2}}\right)\right]\,dx$

What's wrong with this computation?

First, let's all agree that $-\frac{1}{3x^3}$ is an antiderivative for $\frac{1}{x^4}$.

So we compute:

$$\int_{-1}^{1} \frac{1}{x^4} dx \; = \; -\frac{1}{3x^3}\Big|_{-1}^{1} \; = \; \left(-\frac{1}{3}\right) - \left(-\frac{1}{3(-1)}\right) \; = \; -\frac{2}{3}.$$

However, we know that $\frac{1}{x^4}$ is always positive, and so the definite integral should be a *positive* area.

What's wrong here? Is FTC2 not true?