- Reminder: Problem Set 7 is due on Thursday, 30 January.
- Today's lecture will assume you have watched up to and including video 8.7.

For tomorrow's lecture, please watch videos 9.1 through 9.4.

True, false, or can't decide?

We want to find a function H with domain $\mathbb R$ such that H(1) = -2 and such that

$$H'(x) = e^{\sin x}$$
 for all x.

Decide whether each of the following statements is true, false, or you do not have enough information to decide.

Our primary use for FTC1 in this course is to prove FTC2. But it does have some applications of its own as a new differentiation rule.

Problem 1. Find the derivative of $F(x) = \int_0^x \cos^2 t \, dt$. **Problem 2.** Find the derivative of $G(x) = \int_x^7 7te^{t^2} dt$.

(Trickier) Problem 3. Find the derivative of $H(x) = \int_0^{x^2} \frac{\sin t}{1+t} dt$.

Hint. The answer is not $\frac{\sin(x^2)}{1+x^2}$.

Hint 2. Try to express H(x) in terms of $F(x) = \int_0^x \frac{\sin t}{1+t} dt$.

Going further...

(Trickier) Problem 4. Find the derivative of $S(x) = \int_0^x xe^{t^2} dt$.

Problem 5. Find the derivative of
$$F(x) = \int_{x^2}^{\sin x} \arctan t \, dt$$
.

Hint: Try to write F(x) as a sum of two functions whose derivatives you know how to compute.

Okay! We're [trying to be] mathematicians, so it's time to generalize! Why solve one example, when we can solve **all** the examples?

Exercise

Let f, u, v be differentiable functions on \mathbb{R} , and define:

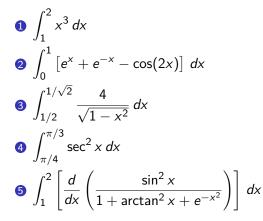
$$F(x) = \int_{u(x)}^{v(x)} f(t) dt.$$

Find a formula for F'(x) in terms of some or all of f, u, v, f', u', v'.

FTC2: Definite integrals

We didn't spend time on these in class, but I'm posting them as extra practice problems.

Compute the following definite integrals:



What's wrong with this computation?

First, let's all agree that
$$-\frac{1}{3x^3}$$
 is an antiderivative for $\frac{1}{x^4}$.

So we compute:

$$\int_{-1}^{1} \frac{1}{x^4} dx = \left. -\frac{1}{3x^3} \right|_{-1}^{1} = \left(-\frac{1}{3} \right) - \left(-\frac{1}{3(-1)} \right) = -\frac{2}{3}.$$

However, we know that $\frac{1}{x^4}$ is always positive, and so the definite integral should be a *positive* area.

What's wrong here? Is FTC2 not true?