MAT137 - Term 2, week 3, lecture 3

- **Reminder:** Problem Set 7 is due on Thursday, 30 January.
- Today's lecture will assume you have watched up to and including video 9.4.

  For next Tuesday's lecture, please watch videos 9.5 through 9.9.

Substitution

Recall from the videos that the technique of substitution is derived from integrating the chain rule.

Here's the chain rule:

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x).$$

Equating antiderivatives of both sides yields:

$$f(g(x)) + C = \int f'(g(x))\, g'(x)\, dx.$$

Substitution notation

$$f(g(x)) + C = \int f'(g(x))\, g'(x)\, dx.$$

When using the substitution rule, we will usually use the notation

$$u = g(x) \quad \text{and} \quad du = g'(x)\, dx = \frac{du}{dx}\, dx.$$

With this notation, the substitution rule says:

$$f(u) + C = \int f'(u)\, du,$$

which is something we essentially already know from FTC1.

This entire process amounts to a change of variables from x to something (called u, usually) that's more convenient for us to integrate with. The tricky part, in general, is choosing the u.

Substitution exercises.

Problem 1. Consider the following integrals. In each case, what should our u be?

1. $\int 7e^{7x+5}\, dx$
2. $\int e^x \cos\left(e^x\right)\, dx$
3. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$
4. $\int \frac{x}{1+x^4}\, dx$

Problem 2. Compute the integral $\int \cot(x) \ln(\sin(x))\, dx$ using substitution.

Problem 3. Compute $\int \frac{(\ln(\ln x))^2}{x(\ln x)}\, dx$.

Problem 4. Compute $\int x\sqrt[3]{x+3}\, dx$.

A different kind of substitution

Problem. Compute the definite integral

$$\int_0^1 \sqrt{1-x^2}\, dx$$

using the substitution $x = \sin\theta$.

Think carefully about how to use this substitution to turn the integral into something like

$$\int_a^b \Big(\text{something}\Big)\, d\theta.$$

Also note that you can use some geometric intuition to check your answer here.