- **Reminder:** Problem Set 7 is due on Thursday, by 11:59pm.
- Today's lecture will assume you have watched up to and including video 9.9.

  For Thursday's lecture, please watch videos 9.10 through 9.12.
- You have some integrals to think about for next class on the last slide.

A theorem about odd functions

Let's use substitution to prove a very useful theorem.

Theorem

Let f be a continuous function defined on all of $\mathbb{R}$. If f is odd, then for any positive real number a,

$$\int_{-a}^{a} f(x)\,dx = 0.$$

1. Convince yourself that this theorem is true by drawing a picture.
2. Make sure you have a clear definition of "odd function" to work with.
3. Express the definite integral above as the sum of two definite integrals in a natural way, then use substitution to prove the theorem.

Integration by parts

Here's the integration by parts formula, as derived in one of the videos:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

We usually use notation similar to what we used with the substitution rule. We let $u = f(x)$ and $v = g(x)$.

Then accordingly we write $du = f'(x)\,dx$ and $dv = g'(x)\,dx$.

With this notation, the formula looks like this:

$$\int u\,dv = uv - \int v\,du.$$

Computation practice: Integration by parts

Use integration by parts (possibly in combination with substitution) to compute the following antiderivatives.

Once you get to a place where you know you can finish, stop. Your goal should always be to reduce the problem to one that you *know you can solve.*

1. $\displaystyle\int x\, e^{-2x}\, dx$
2. $\displaystyle\int \ln x\, dx$
3. $\displaystyle\int \left(\ln x\right)^3 dx$
4. $\displaystyle\int x \arctan x\, dx$
5. $\displaystyle\int \sin\sqrt{x}\, dx$
6. $\displaystyle\int x^2 \arcsin x\, dx$
7. $\displaystyle\int e^{\cos x} \sin^3 x\, dx$
8. $\displaystyle\int e^{ax} \sin(bx)\, dx$

Think about the rest of these for next class.