MAT137 - Term 2, week 4, lecture 2

- **Reminder:** Problem Set 7 is due today, by 11:59pm.
- Today's lecture will assume you have watched up to and including video 9.12.

  For Thursday's lecture, please watch videos 9.15 through 9.17.

Computation practice: Integration by parts

Use integration by parts (possibly in combination with substitution) to compute the following antiderivatives.

Once you get to a place where you know you can finish, stop. Your goal should always be to reduce the problem to one that you *know you can solve.*

1. $\int x\, e^{-2x}\, dx$
2. $\int \ln x\, dx$
3. $\int (\ln x)^3\, dx$
4. $\int x \arctan x\, dx$
5. $\int \sin \sqrt{x}\, dx$
6. $\int x^2 \arcsin x\, dx$
7. $\int e^{\cos x} \sin^3 x\, dx$
8. $\int e^{ax} \sin(bx)\, dx$

A reduction formula

You will to prove a result that allows you to compute the antiderivative of any positive integer power of $\ln x$, by deriving something called a *reduction formula*.

A reduction formula is a formula that expresses one integral in terms of a strictly simpler integral of the same sort.

Problem 1. Let $n > 2$ be an integer. Use integration by parts to come up with a formula of this form:

$$\int (\ln x)^n \, dx = [\text{SOMETHING}] + [\text{CONSTANT}] \int (\ln x)^{n-1} \, dx.$$

Problem 2. Use the formula you derived to compute:

$$\int (\ln x)^3 \, dx \quad \text{which you did earlier, and} \quad \int (\ln x)^{10} \, dx$$

(or at least convince yourself that it's easy to do now).

Integrals of certain combinations of trig functions

In this section we are going to talk about some general methods for integrating certain combinations of trig functions.

There are no new concepts to learn here. We will be using substitution and integration by parts, along with some trig identities.

- The Pythagorean identities:
  - $\sin^2(x) + \cos^2(x) = 1$.
  - $\tan^2(x) + 1 = \sec^2(x)$.
- The angle addition identities:
  - $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$.
  - $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.
- This double angle formula (which is an easy consequence of the last identity above):
  - $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$.

Integrals of certain combinations of trig functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, stop.)

1. $\int \sin^{10} x \cos x \, dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int \sec^{12}(x) \, dx$
4. $\int \cos^2 x \, dx$
5. $\int \sin^4 x \, dx$
6. $\int \tan^7(x) \sec^7(x) \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$