MAT137 - Term 2, week 4, lecture 3

- **Reminder:** Test 3 is next week, on **Thursday**.
- Today's lecture will assume you have watched up to and including video 9.17 (except 9.13 and 9.14).

  For next Tuesday's lecture, please watch video 10.1. You might as well also watch video 10.2, which will carry us through Thursday's lecture.
- NOTE: The conclusion I presented for one of the exercises in class contained an error. Look for the red text below for details.

Integrals of certain combinations of trig functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, stop.)

1. $\int \sin^{10} x \cos x \, dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int \sec^{12}(x) \, dx$
4. $\int \cos^2 x \, dx$
5. $\int \sin^4 x \, dx$
6. $\int \tan^7(x) \sec^7(x) \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Integrals of products of secant and tangent

You did two examples like this on the previous slide. Now let's generalize, and solve *all* the problems!

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

NOTE: The conclusion I presented in class for the second part wasn't correct. I've added a slide after this to explain that.

Integrals of products of secant and tangent

Added after class to explain an error during this lecture.

The answer we gave in class for the second question on the previous slide was that n and m should both be positive and odd.

We arrived at this because a common wrong answer was that n should be odd, but we saw the counterexample $\int \sec^3 x \tan^4 x \, dx$.

That led us to say that m needs to be odd, which is true, but we forgot to fix n. In fact, as long as n is positive and m is odd, we can do the $u = \sec x$ substitution. To see this, let n, m be natural numbers with n positive and m odd. Then:

$$\int \sec^n x \tan^m x \, dx = \int \sec^{n-1} x \tan^{m-1} x \; \sec x \tan x \, dx.$$

Since m is odd we know $m - 1$ is even, meaning that $\tan^{m-1} x$ can be written entirely in terms of $\sec x$. Then the substitution $u = \sec x$ will work.

Integrals of rational functions

1. Let $a \in \mathbb{R}$. Calculate $\displaystyle\int \frac{1}{x+a}\,dx$.
2. Combine this into one rational function: $\displaystyle\frac{2}{x} - \frac{3}{x+3}$.
3. Calculate $\displaystyle\int \frac{-x+6}{x^2+3x}\,dx$.
4. Calculate $\displaystyle\int \frac{1}{x^2+3x}\,dx$.
5. Calculate $\displaystyle\int \frac{1}{x^3-x}\,dx$.