MAT137 - Term 2, week 5, lecture 2

- **Reminder:** Test 3 is **today**
- Today's lecture will assume you have watched up to and including video 10.2.

  For tomorrow's lecture, please watch videos 11.1 and 11.2.

Let A be the region in the first quadrant bounded between the curves with equations $y = x^3$ and $y = \sqrt{32x}$.

Compute the volume of the solid of revolution obtained by rotating A around...

1. ...the x-axis
2. ...the y-axis
3. ...the line $y = -1$

(Just set up the integrals. No need to do the computation once you know you can do it.)

Volumes by “cylindrical shells” (like a volcano!)

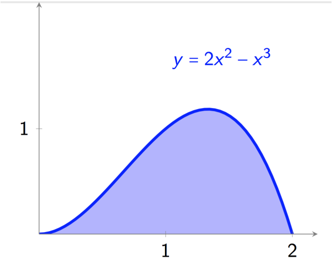
You also saw a specific example of this in the videos. Now let’s do it in general!

Problem. Let $a < b$ be real numbers.
Let f be a continuous, positive function defined on $[a, b]$.
Let A be the region in the first quadrant bounded between the graph of f and the x-axis.

Find a formula for the volume of the solid of revolution obtained by rotating the region A around the $\underline{y\text{-axis}}$.

A volcano!

Consider the region A between the curve $y = 2x^2 - x^3$ and the x-axis in the first quadrant:



- Rotate A around the y-axis, and compute the volume of the resulting solid. (You can use your formula from the last slide!)
- Rotate A around the line $x = -7$, and compute the volume of the resulting solid.

(Just set up the integrals.)

The volume of a doughnut

A bonus exercise that we didn't see in class. It's a good one!

Let A be the region inside the circle with this equation:

$$(x-1)^2+y^2=1.$$

Rotate A around the line with equation $y=4$.
The resulting solid is called a **torus**.

1. Draw a picture and convince yourself that a torus looks like a doughnut.
2. Compute the volume of the torus as an integral using x as the variable (*like a volcano!*)
3. Compute the volume of the torus as an integral using y as the variable (*like a carrot!*)