# MAT137 - Term 2, week 5, lecture 3

- If you showed up to class today, through the snow the morning after a test, you deserve a pat on the back.

- Today's lecture will assume you have watched up to and including video 11.2.

  For next Tuesday's lecture, please watch videos 11.3 and 11.4.

# Quick warm up question

Write a formula for the general term of each of these sequences

1. $\{a_n\}_{n=0}^{\infty} = \{\, 1,\ 4,\ 9,\ 16,\ 25,\ \dots \,\}$
2. $\{b_n\}_{n=1}^{\infty} = \{\, 1,\ -2,\ 4,\ -8,\ 16,\ -32,\ \dots \,\}$
3. $\{c_n\}_{n=1}^{\infty} = \left\{ \frac{2}{1!}, \frac{3}{2!}, \frac{4}{3!}, \frac{5}{4!}, \dots \right\}$
4. $\{d_n\}_{n=1}^{\infty} = \{\, 1,\ 4,\ 7,\ 10,\ 13,\ \dots \,\}$
5. $\{x_n\}_{n=0}^{\infty} = \{\, 0,\ 1,\ 0,\ -1,\ 0,\ 1,\ 0,\ -1,\ 0,\ \dots \,\}$

# True or False?

Let $f$ be a function defined at least on $[1, \infty)$.
We define a sequence by $a_n = f(n)$.
(I suggest drawing a picture at this point.)

Let $L \in \mathbb{R}$.

1. IF $\lim_{x \to \infty} f(x) = L$, THEN $\lim_{n \to \infty} a_n = L$.
2. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{x \to \infty} f(x) = L$.
3. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{n \to \infty} a_{n+1} = L$.

# Definition of limit of a sequence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence, and let $L \in \mathbb{R}$.

Which statements below are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

1. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
2. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n > n_0 \implies |L - a_n| < \varepsilon.$
3. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{R} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
4. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{R},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
5. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| \leq \varepsilon.$
6. $\forall \varepsilon \in (0,1),\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
7. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{\varepsilon}.$
8. $\forall k \in \mathbb{N} > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < k.$
9. $\forall k \in \mathbb{N} > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{k}.$