MAT137 - Term 2, week 6, lecture 1

- Problem Set 8 is available on the course website now. It's due on the Thursday after reading week.

- Today's lecture will assume you have watched up to and including video 11.4.

  For Thursday's lecture, please watch videos 11.5 and 11.6.

Definition of limit of a sequence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence, and let $L \in \mathbb{R}$.

Which statements below are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

1. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
2. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n > n_0 \implies |L - a_n| < \varepsilon.$
3. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{R}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
4. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{R}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
5. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| \leq \varepsilon.$
6. $\forall \varepsilon \in (0,1),\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
7. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \frac{1}{\varepsilon}.$
8. $\forall k \in \mathbb{N} > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < k.$
9. $\forall k \in \mathbb{N} > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \frac{1}{k}.$

Definition of limit of a sequence (continued)

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence, and let $L \in \mathbb{R}$.

Which statements below are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

10. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all but finitely many of the terms of the sequence.
11. $\forall \varepsilon > 0$, the interval $[L - \varepsilon, L + \varepsilon]$ contains all but finitely many of the terms of the sequence.
12. Every interval that contains L must contain all but finitely many of the terms of the sequence.
13. Every open interval that contains L must contain all but finitely many of the terms of the sequence.

True or False – Monotonic sequences vs. functions

Let f be a function defined at least on $[1, \infty)$.
We define a sequence by $a_n = f(n)$.

1. IF f is increasing, THEN $\{a_n\}_{n=0}^{\infty}$ is increasing.
2. IF $\{a_n\}_{n=0}^{\infty}$ is increasing, THEN f is increasing.

(If you think one of them is true, try to prove it.
If you think one of them is false, give a counterexample.)

Monotonicty vs. boundedness vs. convergence

For each of the eight "???" boxes, construct an example sequence if possible.
If any of them is impossible, cite a theorem to justify why.

		convergent	divergent
monotonic	bounded	???	???
	unbounded	???	???
not monotonic	bounded	???	???
	unbounded	???	???