MAT137 - Term 2, week 6, lecture 2

- Problem Set 8 is available on the course website now. It's due on the Thursday after reading week.
- Today's lecture will assume you have watched up to and including video 11.6.

  For tomorrow's lecture, please watch videos 11.7 and 11.8.
- I made an error during this lecture when discussing slide number 3. I have inserted a new slide below explaining this. My apologies for any confusion I may have caused.

Monotonicty vs. boundedness vs. convergence

Last class we ended with this exercise.

The three boxes with red question marks were left empty. Using what you now know, figure out what should be in those boxes.

		convergent	divergent
monotonic	bounded		???
	unbounded	???	
not monotonic	bounded		
	unbounded	???	

True or False?

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

For each of the following: If true, explain why. If false, provide a counterexample.

1. If $\{a_n\}_{n=0}^{\infty}$ is convergent, then it is bounded above.
2. If $\{a_n\}_{n=0}^{\infty}$ is convergent, then it is eventually monotonic.
3. If $\{a_n\}_{n=0}^{\infty}$ diverges and is increasing, then there exists $n \in \mathbb{N}$ such that $a_n > 100$.
4. If $\lim_{n\to\infty} a_n = L$, then $a_n < L + 1$ for all n.
5. If $\lim_{n\to\infty} a_n = L$, then $a_n < L + 1$ for all but finitely many n.
6. If $\{a_n\}_{n=0}^{\infty}$ is non-decreasing and non-increasing, then it is convergent.
7. If $\{a_n\}_{n=0}^{\infty}$ is not decreasing and is not increasing, then it is convergent.
8. If $\lim_{n\to\infty} a_{2n} = L$, then $\lim_{n\to\infty} a_n = L$.

Correcting an error from class today

I made a silly error when discussing the material from the last slide in class on Thursday. The purpose of this slide is to correct it.

In class, I wrote that statement number 5 was false, with $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ as a counterexample.

This wasn't correct, as you can easily verify. All of this should have been said about statement number 4 instead. Statement 4 is false, with the above sequence being a counterexample.

In class, we never actually addressed statement number 5. Statement number 5 is **true**, by the definition of sequence convergence.

Apologies for any confusion I may have caused.

A recursively-defined sequence

Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} a_0 = 1 \\ a_{n+1} = \dfrac{a_n + 2}{a_n + 3}, \quad n \geq 0 \end{cases}$$

Compute a_1, a_2, and a_3.

Is this proof correct?

Let $\{a_n\}_{n=0}^{\infty}$ be the sequence from the previous slide.

Claim.

$\{a_n\}_{n=0}^{\infty}$ converges to $-1+\sqrt{3}$.

Proof.

Let $L = \lim_{n\to\infty} a_n$.

Starting with the recurrence relation and taking limits of both sides, we get

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left[\frac{a_n+2}{a_n+3}\right] \implies L = \frac{L+2}{L+3} \implies L^2+2L-2=0$$

Solving the quadratic yields $L = -1 \pm \sqrt{3}$.

Every term of the sequence is postivie, so L cannot be negative. So we conclude that $L = -1+\sqrt{3}$. □