MAT137 - Term 2, week 7, lecture 1

- Problem Set 8 is due on Thursday.
- Today's lecture will assume you have watched up to and including video 12.6.

  For Thursday's lecture, watch videos 12.7 and 12.8.

Recall the definitions

1. Let f be a bounded, continuous function on $[c, \infty)$.
   How do we define the improper integral
   $$\int_c^{\infty} f(x)\,dx\,?$$
2. Let f be a continuous function on $(a, b]$.
   How do we define the improper integral
   $$\int_a^{b} f(x)\,dx\,?$$

Problem 1. Compute the value of this integral, from the definition.

$$\int_1^\infty \frac{1}{x^2+x}dx$$

Hint: $\dfrac{1}{x^2+x} = \dfrac{(x+1)-(x)}{x(x+1)}$.

Problem 2. Determine whether $\int_7^\infty \cos(x)\,dx$ converges or diverges.

What can you conclude?

Problem 1. Suppose f is continuous on $[1, \infty)$, and that $\lim_{x \to \infty} f(x) = 7$.

What can you conclude about $\int_1^{\infty} f(x)\,dx$? Try to prove your answer.

Problem 2. Suppose f is continuous on $[1, \infty)$, and suppose you know that $\int_1^{\infty} f(x)\,dx$ converges.

What can you conclude about $\lim_{x \to \infty} f(x)$?

What can you conclude? - Follow-up

I said something incorrect at the end of the lecture, about Problem 2 on the previous slide.

We agreed that in Problem 1, the assumption that $\lim_{x\to\infty} f(x) = 7$ implies that $\int_1^\infty f(x)\,dx$ diverges.

The same reasoning led us to say that in Problem 2, **if** $\int_1^\infty f(x)\,dx$ **converges and** $\lim_{x\to\infty} f(x)$ **exists**, then $\lim_{x\to\infty} f(x) = 0$

However, I said a stronger thing is true: the assumption that $\int_1^\infty f(x)\,dx$ converges alone implies that $\lim_{x\to\infty} f(x) = 0$.

This isn't true. There are examples of functions f such that $\int_1^\infty f(x)\,dx$ converges but $\lim_{x\to\infty} f(x)$ does not exist. Try to think of one! It will be tricky at this point, but easier later in the course.

A "simple" integral

I didn't show you this slide in class, but we talked about the idea in it as a follow-up question to the first slide.

Consider the integral $\int_{-1}^{1} \frac{1}{x}\, dx$.

Are either of these arguments correct?

1. $\int_{-1}^{1} \frac{1}{x}\, dx \;=\; \Big[\ln|x|\Big]\Big|_{-1}^{1} \;=\; \ln|1| - \ln|-1| = 0$
2. $\int_{-1}^{1} \frac{1}{x}\, dx = 0$, since $f(x) = \frac{1}{x}$ is an odd function.

Does the integral converge?