MAT137 - Term 2, week 8, lecture 1

- **Reminder:** Problem Set 9 is due next Thursday, 12 March.
- Today's lecture will assume you have watched up to and including video 13.4.

  For Thursday's lecture, watch videos 13.5 through 13.7.

Quick warm-up: Improper integrals!

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$
4. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$
5. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$
6. $\displaystyle\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$
8. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\displaystyle\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

Telescoping series

We want to calculate the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$.

1. Find a formula for the k-th partial sum $S_k = \displaystyle\sum_{n=1}^{k} \frac{1}{n^2 + 2n}$.

   Hint: First write $\displaystyle\frac{1}{n^2 + 2n} = \frac{A}{n} + \frac{B}{n+2}$.

2. Using the definition of a series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}.$$

For convergence, it doesn't matter where you start.

Something we know about sequences is that if you care about the limit of a sequence, it doesn't matter where you start.

In other words, $\{a_n\}_{n=1}^{\infty}$ and $\{a_n\}_{n=17}^{\infty}$ have the same limit (if it exists), and the fact that the first 16 terms of the latter sequence are "missing" doesn't matter.

The same is true of series, in the following sense.

Claim.

Suppose $\{a_n\}_{n=1}^{\infty}$ is a sequence, and $M > 1$ is an integer. Then:

$$\sum_{n=1}^{\infty} a_n \text{ converges} \iff \sum_{n=M}^{\infty} a_n \text{ converges.}$$

Problem. Prove this, using the definition of series convergence.