MAT137 - Term 2, week 8, lecture 2

- **Reminder:** Problem Set 9 is due next Thursday, 12 March.
- Today's lecture will assume you have watched up to and including video 13.7.

  For tomorrow's lecture, watch videos 13.8 and 13.9.

Quick warm-up: More improper integrals!

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{x}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
4. $\displaystyle\int_1^\infty \frac{x+1}{x^3+2}\,dx$
5. $\displaystyle\int_1^\infty \frac{\sqrt{x^2+5}}{x^3+6}\,dx$
6. $\displaystyle\int_1^\infty \frac{x^2+3}{\sqrt{x^5+2}}\,dx$

What is wrong with this calculation? Fix it.

Claim.

$$\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right) = \ln 2$$

"Justification"

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right) &= \sum_{n=2}^{\infty} [\ln(n) - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) \; - \; \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \dots) \; - \; (\ln 3 + \ln 4 + \ln 5 + \dots) \\
&= \ln 2
\end{aligned}$$

Geometric series

Given a real number r (r stands for "ratio"), the series

$$\sum_{n=0}^{\infty} r^n$$

is called a geometric series. These are series we can evaluate explicitly.

Recall the following result from one of the videos:

Theorem

The geometric series $\sum_{n=0}^{\infty} r^n$ *converges if and only if* $|r| < 1$.

In this case, $\displaystyle\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

Geometric series

Compute the values of the following series.

1. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\dots$
2. $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\dots$
3. $\frac{3}{2}-\frac{9}{4}+\frac{27}{8}-\frac{81}{16}+\dots$
4. $1+\frac{1}{2^{0.5}}+\frac{1}{2}+\frac{1}{2^{1.5}}+\frac{1}{2^2}+\frac{1}{2^{2.5}}+\dots$
5. $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{2^{2n+1}}$
6. $\sum_{n=k}^{\infty}x^n$, where $|x|<1$.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series, and let $\{S_n\}_{n=0}^{\infty}$ be its sequence of partial sums.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$.
2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$.
3. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.
4. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.