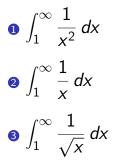
- **Reminder:** Problem Set 9 is due next Thursday, 12 March.
- Today's lecture will assume you have watched up to and including video 13.7.

For tomorrow's lecture, watch videos 13.8 and 13.9.

# Quick warm-up: More improper integrals!



 $\int_{1}^{\infty} \frac{x+1}{x^3+2} dx$  $\int_{1}^{\infty} \frac{\sqrt{x^2+5}}{x^3+6} \, dx$ 6  $\int_{1}^{\infty} \frac{x^2 + 3}{\sqrt{x^5 + 2}} dx$ 

# What is wrong with this calculation? Fix it.

### Claim.

$$\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right) = \ln 2$$

#### "Justification"

$$\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=2}^{\infty} \left[\ln(n) - \ln(n+1)\right]$$
  
=  $\sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1)$   
=  $(\ln 2 + \ln 3 + \ln 4 + ...) - (\ln 3 + \ln 4 + \ln 5 + ...)$   
=  $\ln 2$ 

Given a real number r (r stands for "ratio"), the series

$$\sum_{n=0}^{\infty} r^n$$

is called a  $\underline{geometric\ series}.$  These are series we can evaluate explicitly.

Recall the following result from one of the videos:

#### Theorem

The geometric series 
$$\sum_{n=0}^{\infty} r^n$$
 converges if and only if  $|r| < 1$ .  
In this case,  $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ .

Compute the values of the following series.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$$

$$2 \quad \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \dots$$

$$3 \quad \frac{3}{2} - \frac{9}{4} + \frac{27}{8} - \frac{81}{16} + \dots$$

$$4 \quad 1 + \frac{1}{2^{0.5}} + \frac{1}{2} + \frac{1}{2^{1.5}} + \frac{1}{2^2} + \frac{1}{2^{2.5}} + \dots$$

$$3 \quad \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{2^{2n+1}}$$

$$3 \quad \sum_{n=k}^{\infty} x^n, \text{ where } |x| < 1.$$

## True or False – Series

Let  $\sum a_n$  be a series, and let  $\{S_n\}_{n=0}^{\infty}$  be its sequence of partial sums. **1** IF the series  $\sum_{n=1}^{\infty} a_n$  is divergent, THEN  $\exists n \in \mathbb{N}$  such that  $a_n > 100$ . **2** IF the series  $\sum a_n$  is divergent, THEN  $\exists n \in \mathbb{N}$  such that  $S_n > 100$ . **3** IF the series  $\sum_{n=1}^{\infty} a_n$  converges, THEN the series  $\sum_{n=1}^{\infty} a_n$  converges to a smaller number. n = 100**4** IF the series  $\sum_{n=1}^{\infty} a_n$  converges, THEN the sequence  $\{S_n\}_{n=0}^{\infty}$  is eventually monotonic.