MAT137 - Term 2, week 8, lecture 3

- **Reminder:** Problem Set 9 is due next Thursday, 12 March.
- Today's lecture will assume you have watched up to and including video 13.9.

  For next Tuesday's lecture, watch videos 13.10 through 13.12.

Quick warp-up: Convergent or divergent?

1. $$\sum_{n=0}^{\infty} \frac{1}{2^n}$$
2. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$$
3. $$\sum_{n=1}^{\infty} \frac{1}{2^{n/2}}$$
4. $$\sum_{n=5}^{\infty} \frac{3^n}{2^{2n+1}}$$
5. $$\sum_{n=3}^{\infty} \frac{3^n}{1000 \cdot 2^{n+2}}$$
6. $$\sum_{n=0}^{\infty} (-1)^n$$

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series, and let $\{S_n\}_{n=0}^{\infty}$ be its sequence of partial sums.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$.
2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$.
3. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.
4. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

True or False – Series, Part 2

Let $\sum_{n=0}^{\infty} a_n$ be a series, and let $\{S_n\}_{n=0}^{\infty}$ be its sequence of partial sums.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

6. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing, THEN $\forall n \geq 0,\ a_n > 0$.

7. IF $\lim_{n\to\infty} a_n = 0$, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

8. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.

True or False – The Necessary Condition

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^{\infty} a_n$ is convergent.
2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_n^{\infty} a_n$ is divergent.
3. IF $\sum_n^{\infty} a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.
4. IF $\sum_n^{\infty} a_n$ is divergent, THEN $\lim_{n\to\infty} a_n \neq 0$.

From the geometric series formula, we know that when $|x| < 1$, we can expand the function $f(x) = \dfrac{1}{1-x}$ as a series:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \text{for } |x| < 1$$

Find similar ways to write the following functions as series:

1. $g(x) = \dfrac{1}{1+x}$
2. $h(x) = \dfrac{1}{1-x^2}$
3. $k(x) = \dfrac{1}{2-x}$

TRUE OR FALSE!?

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^{\infty} a_n$ is convergent.