MAT137 - Term 2, week 9, lecture 1

- **Reminder:** Problem Set 9 is due on Thursday.
- Today's lecture will assume you have watched up to and including video 13.12.

  For Thursday's lecture, watch videos 13.13 and 13.14.
- You have some series to think about for next class. See the last slide.

Quick warm-up – The Necessary Condition

True or False?

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^\infty a_n$ is convergent.
2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_n^\infty a_n$ is divergent.
3. IF $\sum_n^\infty a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.
4. IF $\sum_n^\infty a_n$ is divergent, THEN $\lim_{n\to\infty} a_n \neq 0$.

Pep talk

For today's class, you watched videos about:

- the integral test for series,
- the basic comparison test for series,
- the limit comparison test for series.

On PS9, you will explore, in detail, the connection between improper integrals like $\int_1^\infty f(x)\,dx$ and series like $\sum_{n=1}^{\infty} f(n)$.

So today, we're going to concentrate on building your intuition for using the convergence tests.

More quick warm-up

For which values of $p \in \mathbb{R}$ do these series converge?

1. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{p^n}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^p}$
3. $\displaystyle\sum_{n=0}^{\infty} p^n$
4. $\displaystyle\sum_{n=0}^{\infty} n^p$

More quick warm-ups – Convergent or divergent?

1. $\sum_{n}^{\infty} \dfrac{n^{10} + 17n^{7} + 3}{n^{11}}$

2. $\sum_{n}^{\infty} \dfrac{\sqrt[3]{n^2 + 1} + 1}{\sqrt{n^4 + n} + n + 1}$

SURPRISE!

TRUE OR FALSE!?

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

IF $\lim_{n\to\infty} a_n = 0$,

THEN $\sum_{n}^{\infty} a_n$ is convergent.

Slower questions – Convergent or divergent?

Think about the rest of these for next class.

1. $\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$
2. $\sum_{n}^{\infty} \frac{(\ln n)^{20}}{n^2}$
3. $\sum_{n}^{\infty} \sin^2 \frac{1}{n}$
4. $\sum_{n}^{\infty} \frac{1}{n(\ln n)^3}$
5. $\sum_{n}^{\infty} \frac{1}{n \ln n}$
6. $\sum_{n}^{\infty} e^{-n^2}$