MAT137 - Term 2, week 10, lecture 1

- These slides are representative of what I would have brought to class with me, were in-person not lectures not cancelled. Please feel free to work on these questions, as I think you will find them useful practice.

  It is likely that Asif will cover some of these questions during his online lectures. For those or any other questions from these slides, please feel free to ask about them on Piazza or during online office hours.

- This lecture assumes you have watched up to and including video 14.2.

  For the next lecture, watch videos 14.3 and 14.4.

Find the interval of convergence (i.e., not just the *radius* of convergence) of each of the following power series:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2\, 2^{2n+1}}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{42^n} x^n$
4. **Tricky!** $\displaystyle\sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!} x^n$

What can you conclude?

Consider the power series $\sum_n a_n x^n$. Do not assume $a_n \geq 0$.

In each case, may the given series be absolutely convergent (AC)? conditionally convergent (CC)? divergent (D)? all of them?

IF	$\sum_n a_n 3^n$ is ...	AC	CC	D
THEN	$\sum_n a_n 2^n$ may be ...			
	$\sum_n a_n (-3)^n$ may be ...			
	$\sum_n a_n 4^n$ may be ...			

Using the geometric series, we know how to write the function $F(x) = \frac{1}{1-x}$ as a power series centered at 0:

$$F(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

Write the following functions as power series centered at 0:

1. $f(x) = \frac{1}{1+x}$
2. $g(x) = \frac{1}{1-x^2}$
3. $h(x) = \frac{1}{2-x}$
4. $G(x) = \ln(1+x)$

Challenge

We want to calculate the value of $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

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1. What is the value of the sum $\sum_{n=0}^{\infty} x^n$, when $|x| < 1$?
2. What is the relation between $\sum_{n}^{\infty} x^n$ and $\sum_{n}^{\infty} n\, x^{n-1}$?
3. Compute the value of the sum $\sum_{n=1}^{\infty} n\, x^{n-1}$.
4. Compute the value of the sum $\sum_{n=1}^{\infty} n\, x^n$.
5. Compute the value of the original series.