MAT137 - Term 2, week 10, lecture 3

- These slides are representative of what I would have brought to class with me, were in-person not lectures not cancelled. Please feel free to work on these questions, as I think you will find them useful practice.

  It is likely that Asif will cover some of these questions during his online lectures. For those or any other questions from these slides, please feel free to ask about them on Piazza or during online office hours.

- This lecture assumes you have watched up to and including video 14.4.

  For the next lecture, watch videos 14.5 and 14.6.

The definitions of Taylor polynomial

Let f be a function defined at and near $a \in \mathbb{R}$. Let $n \in \mathbb{N}$.
Let P_n be the n-th Taylor polynomial for f at a.

Which ones of these is true?

1. P_n is an approximation for f of order n near a.
2. f is an approximation for P_n of order n near a.
3. $\lim_{x \to a} [f(x) - P_n(x)] = 0$
4. $\lim_{x \to a} \dfrac{f(x) - P_n(x)}{(x-a)^n} = 0$
5. $\exists$ a function R_n s.t. $f(x) = P_n(x) + R_n(x)$ and $\lim_{x \to a} \dfrac{R_n(x)}{(x-a)^n} = 0$.
6. $f^{(n)}(a) = P_n^{(n)}(a)$
7. $\forall k = 0, 1, 2, \ldots, n, \quad f^{(k)}(a) = P_n^{(k)}(a)$
8. If x is close to a, then $f(x) = P_n(x)$.

True or False – C^n functions

1. If f and g are C^n functions, then $f+g$ is a C^n function.
2. f is continuous if and only if f is C^0.
3. f is differentiable if and only if f is C^1.
4. If f is a C^n function, then f is a C^{n-1} function.

An explicit equation for Taylor polynomials

1. Find one polynomial P of degree 3 that satisfies

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

2. Find *all* polynomials P that satisfy

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

3. Find a polynomial P of smallest possible degree that satisfies

$$P(0) = A, \quad P'(0) = B, \quad P''(0) = C, \quad P'''(0) = D$$

4. Let f be a C^3 function. Find an explicit formula for the 3-rd Taylor polynomial for a function f at 0.

5. Let f be a C^∞ function, and let $n > 0$. Find an explicit formula for the n-th Taylor polynomial for a function f at 0.

Examples

Use your new formula to compute the Taylor polynomials for these functions at 0:

1. $f(x) = e^x$
2. $g(x) = \sin x$
3. $h(x) = \cos x$

Approximating functions

Which one of the following functions is a better approximation for $F(x) = e^x$ near 0?

1. $f(x) = 1 + x + \dfrac{x^2}{2}$
2. $g(x) = \sin x + \cos x + x^2$
3. $h(x) = e^{-x} + 2x$