MAT137 - Term 2, week 11, lecture 1

- These slides are representative of what I would have brought to class with me, were in-person not lectures not cancelled. Please feel free to work on these questions, as I think you will find them useful practice.

  It is likely that Asif will cover some of these questions during his online lectures. For those or any other questions from these slides, please feel free to ask about them on Piazza or during online office hours.

- This lecture assumes you have watched up to and including video 14.6.

  For the next lecture, watch videos 14.7 and 14.8.

Why do we write Taylor polynomials like that?

Consider this polynomial:

$$f(x) = -\frac{293}{8} + 29x + \frac{13}{4}x^2 - 3x^3 + \frac{3}{8}x^4$$

Without using a calculator, try to compute $f(3)$, $f'(3)$, $f''(3)$, $f'''(3)$, $f^{(4)}(3)$. (Stop after two minutes.)

Now consider *this* polynomial:

$$g(x) = 29 + 8(x-3) - \frac{7}{2}(x-3)^2 + \frac{9}{6}(x-3)^3 + \frac{9}{24}(x-4)^4$$

Do the same for g.

Which one was easier? Discuss why it was easier.

Taylor polynomial of a polynomial

Let $f(x) = x^3$.

1. Write the 2nd Taylor polynomial P_2 for f at 0
   Verify that it is the correct answer using each of the three definitions of a Taylor polynomial.
2. Write the 2nd Taylor polynomial for f at 1.
3. Write the 3rd Taylor polynomial for f at 1

Taylor series not at 0

Write the Taylor series...

1. ...for $f(x) = e^x$ at $a = 2$.
2. ...for $g(x) = \sin x$ at $a = \frac{\pi}{4}$.
3. ...for $H(x) = \frac{1}{x}$ at $a = 3$.

You can do these problems in two ways:

- Method 1: Compute the first few derivatives, guess the pattern (and prove it by induction).
- Method 2: Use the substitution $u = x - a$ and reduce it to an old problem (without computing any derivatives).

More power series

Write the following functions as power series centered at 0:

1. $g(x) = \ln(1 + x)$
2. $h(x) = \arctan x$
3. $f(x) = \sin^2 x$.

Hint: For each of the first two, take one single derivative. Then stop to think.

For the third one, *do not* try to multiply power series together. (It's possible, but harder than necessary here.)