

- These slides are representative of what I would have brought to class with me, were in-person not lectures not cancelled. Please feel free to work on these questions, as I think you will find them useful practice.

It is likely that Asif will cover some of these questions during his online lectures. For those or any other questions from these slides, please feel free to ask about them on Piazza or during online office hours.

- This lecture assumes you have watched up to and including video 14.10.

For the next lecture, watch all the remaining videos on playlist 14. I'm combining all the applications into one big set of slides.

# Taylor series gymnastics

**Problem 1.** Write the following functions as power series centered at 0. You don't need to compute any derivatives to do these.

①  $f(x) = \frac{x^2}{1+x}$

*Hint:*  $f(x) = x^2 \cdot \frac{1}{1+x}$ .

②  $f(x) = x^5 \log(1+x^3)$

*Hint:* This is pretty easy. Don't overthink.

③  $f(x) = \sin(2x^3)$

*Hint:* This is also easy. Don't overthink.

④  $f(x) = \ln \frac{1+x}{1-x}$

*Hint:* Use logarithm rules.

⑤  $f(x) = e^x(1-x^2)$

*Hint:* Combine two series you already know.

**Problem 2.** For each function in previous question, compute  $f^{(2019)}(0)$ .

**Problem.** What function is this the Taylor series of?

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)(n+2)} x^{n+2}$$

*Hint:* First, think about whether this series looks like it has been differentiated or integrated. If so, what series was differentiated or integrated to obtain it?

**Follow-up problem.** What is the value of the following series?

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)(n+2) 2^n}$$

**Problem.** Find the **first four non-zero terms** of the Maclaurin series of these functions:

①  $f(x) = e^x \sin x$

②  $g(x) = e^{\sin x}$

*Hint:* Treat the power series the same way you would treat a polynomial.

## Follow-up problems.

We just found out that:

$$e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \dots$$

**Problem 1.** What is  $g^{(4)}(0)$ ?

**Problem 2.** What is  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1 - x}{x^2 + x^3}$ ?

**Problem 3.** What is  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1 - x}{\cos x - 1}$ ?

**Problem 4.** What is  $\lim_{x \rightarrow 0} \frac{x^2(e^{\sin x} - 1 - x)}{(\cos x - 1)^2}$ ?

Do you understand the power you now have? Do you see the matrix code?

