Welcome back to MAT137 - Calculus with proofs!

- Assignment 6 (unit 7) due on January 28.

- **Before next class:**
  - Watch videos 7.3, 7.4.
  - Download next class slides.
    No need to look at them.

Warm-up: sums

Compute

1. $\sum_{i=2}^{4}(2i+1)$
2. $\sum_{j=2}^{4}(2i+1)$

Write these sums with Σ notation

1. $1^5 + 2^5 + 3^5 + 4^5 + \ldots + N^5$
2. $\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots + \frac{1}{100!}$
3. $\frac{2x^2}{4!} + \frac{4x^3}{5!} + \frac{6x^4}{6!} + \ldots + \frac{198x^{100}}{102!}$

Re-writing sums

1. $\displaystyle\sum_{i=1}^{100} \tan i \; - \; \sum_{i=1}^{50} \tan i \; = \; \sum_{\square}^{\square} \square$

2. $\displaystyle\sum_{i=1}^{N} (2i-1)^5 \; = \; \sum_{i=0}^{N-1} \square$

3. $\displaystyle\left[\sum_{k=1}^{N} x^k\right] + \left[\sum_{k=0}^{N} k\, x^{k+1}\right] \; = \; \left[\sum_{k=\square}^{\square} \square\, x^k\right] + \square$

Hint: Write out the sums on the left hand side first, simplify if possible, then write them back into sigma notation.

Telescopic sum

- Calculate the exact value of

$$\sum_{i=1}^{137} \left[\frac{1}{i} - \frac{1}{i+1} \right]$$

Hint: Write down the first few terms.

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Hint: Write down the first few terms.

- Calculate the exact value of

$$\sum_{i=1}^{10{,}000} \frac{1}{i(i+1)}$$

Fubini-Tonelli warm up

Consider the sum

$$\sum_{i=1}^{4} \sum_{k=1}^{i} \frac{e^i}{k^2}$$

- Write out this sum.
- Rearrange the sum so that all the terms with denominator 1^2 are listed first, then the ones with denominator 2^2, etc.
- Then rewrite in the following form and decide what to write instead of each "$\boxed{?}$":

$$\sum_{k=\boxed{?}}^{\boxed{?}} \sum_{i=\boxed{?}}^{\boxed{?}} \frac{e^i}{k^2}$$

Fubini-Tonelli

- $A_{i,k}$ is a function of 2 variables. For example, $A_{i,k} = \dfrac{e^i}{k^2}$.
- Decide what to write instead of each "$\boxed{?}$" so that the following identity is true:

$$\sum_{i=1}^{N} \sum_{k=1}^{i} A_{i,k} = \sum_{k=\boxed{?}}^{\boxed{?}} \sum_{i=\boxed{?}}^{\boxed{?}} A_{i,k}$$