MAT347Y1 HW17 Marking Scheme

Friday, March 18

Total: 26 points.

13.4.2: 4 points. Remember that whether a polynomial is irreducible depends on the field you're looking at (you can't just use irreducibility over $\mathbb{Q}$).

13.4.5: 6 points.

- (2) Every irreducible polynomial splits completely $\Rightarrow K$ is a splitting field (consider the minimal polynomials of the generators)
- (4) The other direction. This problem was supposed to test your ability to apply Theorems "A" (8) and "B" (27), but there is indeed an easy way out using Alfonso's Theorem 5.11. Since this theorem was proven in class, I gave you full marks if you used it, but it would be good practice to attempt this problem again without it. (Note that the equivalence you use corresponds to Theorem 14.13 in the textbook, so you should definitely be able to solve a chapter 13 problem without it).

14.2.7: 7 points. I assume the discussion in the book as a given (so if you reprove what's already stated, I didn't mark it). Thus all I'm looking for is:

- (1) Statement that Galois extensions $\Leftrightarrow$ normal subgroups
- (5) Finding all normal subgroups of the Galois group (yes, this is actually a group theory question. Please justify your solution)
- (1) Identify corresponding subextensions. To check your work: you should get $\mathbb{Q}(\sqrt{2})$, $\mathbb{Q}(\sqrt{-2})$, $\mathbb{Q}(i)$ (corresponding to index 2 subgroups), $\mathbb{Q}(i, \sqrt[4]{2})$ (corresponding to the center), and $\mathbb{Q}(i, \sqrt{2})$ (easy to check). The rest fit into two conjugate pairs and one conjugate quadruple.

An alternate solution would be to figure out polynomials for which $\mathbb{Q}(i, \sqrt{2})$ and $\mathbb{Q}(i, \sqrt[4]{2})$ are splitting fields over $\mathbb{Q}$, and then since any other subfield is a simple extension, you can check whether it contains all roots of the minimal polynomial of its generator. This could be more or less work depending on how well you remember your group theory.

14.2.13: 4 points. There are surprisingly many distinct (correct) solutions to this question!

14.2.14: 5 points.

- (1) The extension has degree 4
- (2) The extension is Galois (in particular, why is $\sqrt{2-\sqrt{2}}$ in the field?)
- (2) Find a degree 4 automorphism