1. Sheaves on topological spaces

1.1. Sheaves of vector spaces, stalks. Examples: skyscrapers, constant and locally constant sheaves. Locally constant sheaves and representations of π_1. Pull-back, push-forward and proper push-forward. The case of open and closed embeddings. Internal Hom and tensor product functors.

1.2. Reminder on derived categories and derived functors. Injective sheaves. Derived push-forward. Flabby sheaves. Derived proper push-forward and c-soft sheaves. Derived internal Hom and tensor product functors, flat sheaves. The functor $f^!$. Relation between functors and the six functor formalism. [1]

2. Constructible sheaves on algebraic varieties

2.1. Results on morphisms of algebraic varieties. Pull-backs and push-forwards under morphisms. Stratifications incl. Whitney stratification. Constructible sheaves. Constructible derived category.

2.2. Six functors preserve constructibility. Results on cohomology of constructible sheaves. Verdier duality.

3. Intersection homology and perverse sheaves

3.1. Definition of intersection homology. Examples of computations. Intersection homology with coefficients in a local system. Properties of intersection homology. Intersection cohomology sheaves.

3.2. Triangulated categories and t-structures. The perverse t-structure on the constructible derived category. Intersection cohomology sheaf as an example of a perverse sheaf. Properties of the category of perverse sheaves, including the classification of simples.

3.3. Connection to D-modules. The sheaf of differential operators and definition of D-modules. Example: vector bundles with flat connections. Holonomic D-modules. The statement of Riemann-Hilbert correspondence.

4. Decomposition theorem

4.1. Statements of the Deligne theorem on the degeneration of the Leray spectral sequence and of the BBD decomposition theorem and examples of computation. Special cases of small and semi-small maps.

4.2. Background (from topology of smooth projective varieties and Hodge theory).

4.3. A (sketch of) proof of the decomposition theorem for semi-small maps.

[1] This is definitely more than a single lecture.