

Original hints for problem (1):

(a): use $\phi_{t+r}(p) = \phi_r(\phi_t(p))$. (b): use (a). (c): use the fundamental theorem of ODEs on manifolds. (d): you may use the fact that a proper injective immersion is an embedding.

From email of 21 Nov 2016:

Question (1) (a) is easy but it requires you to come to terms with relevant definitions. Here are two facts that you're allowed to use without justification.

- Given a flow ϕ_t ,
its trajectories are the curves $t \mapsto \phi_t(p)$,
and its velocity vector field consists of the tangents to these curves
(which we can denote $(d/dt) \phi_t(p)$).

- Given a map $F: M \rightarrow N$ and a curve $\gamma(t)$ in M ,
the pushforward F_* takes the tangent to the curve $\gamma(t)$ at time t_0
to the tangent to the curve $f(\gamma(t))$ at time t_0 .

(The first of these facts follows from the definition of "velocity vector field". The second of these facts follows from the definition of "pushforward" and from the definition of "tangent to a curve".)

24 Nov'16: the original hint for (1) (c) is actually not necessary; you can just use the definition of a (global) flow.

Original hints for problem (2): use charts. Note that $1/(1+|u|^2) = 1 - |u|^2 + \text{higher order terms}$.