MAT157 – ANALYSIS I, 2018–19. ASSIGNMENT 2.

- Read Chapter 2 of Spivak's book.
- Read the beginning and the end of Chapter 8 of Spivak's book:
  - Pages 133–135, until before Theorem 7-1; and, then,
  - Pages 138–139, starting with the discussion above Theorem 2 (which begins right after the end of the proof of Theorem 7-3), until the end of the chapter.
- Read your class notes.
- Read the handout about the least upper bound property of the real numbers.

Clear solutions to the following problems are due in class on Tuesday September 25th.

(1) Define the Fibonacci sequence $a_1, a_2, a_3, \ldots$ recursively by $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 3$. Prove, by induction on $n$, that

$$a_n = \frac{1}{\sqrt{5}} \left( \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right).$$

(2) Show that, if both $\ell$ and $\ell'$ are greatest lower bounds of a set $A$, then $\ell = \ell'$.

(3) Let $a < b$.
- Prove that if $t \in [0,1]$ and $x = (1-t)a + tb$ then $x \in [a,b]$.
- Prove that if $x \in [a,b]$ then there exists $t \in [0,1]$ such that $x = (1-t)a + tb$.

(4) Submit a solution to one of the recommended problems from Spivak's Chapter 8 that are listed below.

Recommended problems from Spivak's Chapter 2:

- 3 (a), (b), (c), (d), (e)(i),(ii) (binomial coefficients, Pascal's triangle).
- 5(a),(b) (geometric sum).
- 12, 13 (rational and irrational numbers).
- 23, 2nd part ($(a^n)^m = a^{nm}$).

Recommended problems from Spivak's Chapter 8:

- 1 (ii), (iii), (viii) (sup/inf of concrete sets);
- 2(a),(b) ($\inf A = -\sup(-A) = \sup\{\text{lower bounds of } A\}$);
- 5(a),(b) (rational numbers are dense);
- 12(a)(b) (if $A \leq B$ then $\sup A \leq \inf B$);
- 17(a)(b) (characterization of $(-\infty, \alpha)$).