MAT157 – Analysis I, 2018–19. Assignment 3.

Read Chapters 3 and 4 of Spivak's book. Review your class notes. Identify what you don't understand, and come to ask about it. Clear solutions to the following problems are due in class on Tuesday October 2nd.

- (1) ("Lagrange interpolation formula".) Let  $a_1, \ldots, a_n$  and  $b_1, \ldots, b_n$  be real numbers. Assume that  $a_1, \ldots, a_n$  are distinct (i.e.,  $a_i \neq a_j$  whenever  $i \neq j$ ).
  - (a) For each  $i \in \{1, ..., n\}$  find a polynomial  $p_i$  of degree n-1 such that  $p_i(a_i) = 1$ and such that  $p_i(a_j) = 0$  whenever  $j \neq i$ . (Hint: the product  $\prod_{\substack{j \in \{1,...,n\} \setminus \{i\}}} (x-a_j)$ is a polynomial in x of degree n-1, which is zero when  $x = a_j$  for  $j \neq i$  and is non-zero when  $x = a_i$ .)
  - (b) Find a polynomial p of degree  $\leq n-1$  such that  $p(a_i) = b_i$  for all i. (Hint: use the  $p_i$ s from part (a).)
- (2) A function  $f : \mathbb{R} \to \mathbb{R}$  is even if f(-x) = f(x) for all x and odd if f(-x) = -f(x) for all x. For example,  $f(x) = x^2$  is even and f(x) = 7x is odd.
  - (a) Express each of the following functions f as a sum of functions f = E + O where E is even and O is odd. Use as simple expressions as possible for E and O.
    (i) f(x) = (x + 2)<sup>3</sup>.
    - (ii)  $f(x) = \tan(x + \pi/4)$
  - (b) Given any function  $f : \mathbb{R} \to \mathbb{R}$ , show that the function (f(x) + f(-x))/2 is even and that the function (f(x) - f(-x))/2 is odd. Conclude that f can be written as the sum of an even function and an odd function.
- (3) Let A, B, C, D, E be real numbers such that A is positive and C is negative. The set of points (x, y) that satisfy Ax<sup>2</sup> + Bx + Cy<sup>2</sup> + Dy + E = 0 is (usually) a hyperbola.
  (a) Show this.
  - (b) Why "usually"? Can this set be something other than a hyperbola?

(Hint: complete the square; for example, if  $A \neq 0$  write  $Ax^2 + Bx$  as  $A(x + \frac{B}{2A})^2 +$  a constant.)

Solve the following questions but do not hand in your solutions:

- Spivak Chapter 3 Question 1 parts (i), (ii) (composition).
- Spivak Chapter 3 Question 3 parts (i), (ii) (domains of functions).
- Spivak Chapter 3 Question 8 (fractional linear transformation).
- Spivak Chapter 3 Question 9 parts (a), (b) (characteristic function of a set).
- Spivak Chapter 3 Question 12 parts (a),(b),(c) (sum/product/composition of even/odd functions).
- Spivak Chapter 4 Question 11 parts (i)–(iv) (graph of an even/odd/non-negative/periodic function).
- Spivak Chapter 4 Question 14 parts (i)–(iv) (graphs of  $x \mapsto f(x) + c$ , f(x+c), cf(x), f(cx)).
- Spivak Chapter 4 Question 1 parts (iv), (viii) (draw sets given by inequalities; name them as intervals or as unions of intervals).
- Spivak Chapter 4 Question 17 Parts (i), (ii), (v) (graphs of  $\lfloor x \rfloor, x \lfloor x \rfloor, \lfloor 1/x \rfloor$ ).