MAT157 – Analysis I, 2018–19. Assignment 8.

Please read Chapter 11 ("Significance of the derivative") of Spivak's book. Please also read the two(!) handouts on applications of the derivatives on the course website. Clear solutions to the following problems are due in the tutorial on Thursday November 22nd.

(1) Consider the following two functions, each on the given closed interval.

$$t \mapsto t^3 - 7t^2 + 8t + 5 \quad \text{for } 0 \le t \le 4$$

and

$$t \mapsto \frac{t}{t^2 - 1} \quad \text{for } 0 \le t \le 5.$$

For each of these two functions, do the following.
   (a) Compute the values of the function at the points where its derivative vanishes and at the endpoints of the interval.
   (b) Find the minimum and maximum values of the function.
   (c) Find the local minimum points and local maximum points of the function.
   (d) Sketch a graph of the function.

(2) Spivak Chapter 11 Problem 15 (Page 207). (Ladder passing corner.)

(3) Let f be a function that is defined on some open interval that contains x. We say that f is **Lipschitz at** x if there exists a constant $K > 0$ such that $|f(x)-f(y)| \le K|x-y|$ for all y in some open interval that contains x.
   (a) Suppose that f is differentiable at x. Show that f is Lipschitz at x.
   (b) Suppose that f is Lipschitz at x. Show that f is continuous at x.

(4) Spivak Chapter 11 Problem 53 (Page 213). ($f'(0)$ when $f(x) = g(x)/x \ldots$)

Please solve most of the following questions. Do not hand in your solutions.

- Spivak Chapter 11 Problem 4 (a,b) (Page 206). (L^2 and L^1 distances to diagonal.)
- Spivak Chapter 11 Problem 7 (Page 207). (Light reflection.)
- Spivak Chapter 11 Problem 10 (Page 207). (Isoperimetric-type inequality for rectangles.)
- Spivak Chapter 11 Problem 21 (Page 208). (Information on f given graph of f'.)
- Spivak Chapter 11 Problem 32 (Page 209). (Affect of gravitation.)
- Spivak Chapter 11 Problem 43 (Page 211). (Properties of f when $f'(x) = 1/x$.)
- Spivak Chapter 11 Problem 47 (Page 212). (Estimate $\sqrt{66}$.)
- Spivak Chapter 11 Problem 66 (Page 215). (Local minimum with $f'' = 0$.)
- Spivak Chapter 11 Problem 67 (Page 215). (Oscillating function sandwiched between increasing functions.)