MAT157 – Analysis I, 2018–19. Assignment 9.

Please read Chapter 12 ("Inverse functions") of Spivak's book. Clear solutions to the following problems are due in the tutorial on Thursday November 29nd.

(1) (a) Suppose that a function f is twice differentiable at point x_0 and that $f'(x_0) = 0$ and $f''(x_0) > 0$. Prove that f has a strict local minimum at x_0 .

Please do this by copying, with whatever minor adjustments are needed, one of the proofs in the 2nd handout on applications of derivatives. Do not give more details than I gave in the handout. Make sure that you know how to justify each step in that proof, but do not write these justifications in your solution.

- (b) Conclude that, if a function f is twice differentiable at point x_0 and has a local maximum point at x_0 , then $f''(x_0) \leq 0$.
- (2) Let $h: \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies $h'(x) = 1 + \sin^2(1 e^x)$ for all x and h(0) = 7. (a) In 1–3 sentences, explain why h is one-to-one. (b) Let $g = h^{-1}$ (the inverse function). Find g'(7). Show your work.
- (3) (a) Spivak Chapter 12 Problem 2 (Page 240) (graph of inverse of a function f that is increasing/decreasing and positive/negative); assume that f is defined on all of R.
 - (b) Give three examples of continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that $f^{-1} = f$. Give a 1–3 sentence informal comment about such functions. Hint: read Spivak Chapter 12 Problem 24 (Page 242).

Please solve the following questions. Do not hand in your solutions.

- Solve the exercise that we gave in class on Nov.22nd about finding inverse functions after possibly restricting the domain of the original function.
- Spivak Chapter 12 Problem 6 (Page 240) (invert Möbius transformations).
- Spivak Chapter 12 Problem 8 (Page 240) (given f', find relation between $g := f^{-1}$ and its derivative).
- Spivak Chapter 12 Problem 22 (Page 242) (derivative of the inverse of a function, assuming properties of the function).
- Spivak Chapter 11, Problems 51 and 52 (usages of L'Hôpital's rule).
- Read Spivak Chapter 11 Questions 54, 55, and 56. Choose *one* of the items or steps in one of these questions; think about it's statement; find an example where its assumptions (and hence its conclusion) hold.