MAT157 – Analysis I, 2018–19. Assignment 14.

Please read Spivak Chapter 18 ("The logarithm and exponential functions"), the online handout on the logarithm and exponential functions, and Spivak Chapter 19 ("Integration in elementary terms"), including the appendix to Chapter 19 ("The cosmopolitan integral"). Clear solutions to the following problems are due in class on Tuesday March 5th.

(1) A question from a past final exam at UTSG:
Show that the equation $f(x) = x^2 - x\sin x - \cos x = 0$ has exactly two real solutions.

(2) Sketch the graphs of the following two functions. Have each graph show the domain of definition, the minimum and maximum points and values of the function, the regions where the function increases or decreases, and the behaviour of the function near points where it is not defined (if any) and as $x \to \pm\infty$; show the the x– and y–intercepts (if any). Show your work.

(i) The hyperbolic sine function: $\sinh x = \dfrac{e^x - e^{-x}}{2}$.

(ii) The hyperbolic cosine function: $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

(3) (a) Show that $\cosh^2 - \sinh^2 \equiv 1$.

(b) The hyperbolic sine function has an inverse, which is a function $\sinh^{-1}$ that is defined on all of $\mathbb{R}$. Explain why this is true, and find the derivative $(\sinh^{-1})'(x)$.

(4) Any function that is obtained from $\sin(x)$ and $\cos(x)$ by multiplications, additions, and quotients, has an elementary primitive. Indeed, the evil substitution $t = \tan\frac{x}{2}$ transforms the integral of this function into the integral of a rational function. Find $\int \frac{dx}{1+\sin x}$ using this method. (Hint: see Chapter 19 Problem 12.)

Solve the following questions from Spivak's book. Do not hand in your solutions.

- Chapter 18 Problem 1 (p.351): three items of your choice. (Differentiate functions involving powers.)
- Chapter 18 Problem 3: find $\int_a^b \frac{f'(t)}{f(t)}$, for a positive differentiable function f on $[a, b]$.
- Chapter 18 Problem 6: one item of your choice. (Limits of powers.)
- Ch. 18 Problem 24: functions f such that $\int_0^x f(t)dt = e^x$, or such that $\int_0^{x^2} f = 1 - e^{2x^2}$.