MAT157 – Analysis I, 2018–19. Assignment $16\frac{3}{4}$.

Please read Spivak Chapter 24 ("Uniform convergence and power series").

Here are some recommended problems.

- Spivak Chapter 24 Problem 3 (page 517): do two parts. (Find a Taylor series.)

- Spivak Chapter 24 Problem 4 (page 518): do one part.
  (Recognize the limit of a power series.)

- Spivak Chapter 24 Problem 5 (page 518): do one part.
  (Use power series to evaluate an infinite sum of numbers.)

- Spivak Chapter 24 Problem 15 (page 521).
  (Interval of convergence of Tayler series for $\log(1 - x)$ and for $\log(\frac{1+x}{1-x})$.)

- Consider functions $f_n \colon [0, 1] \to \mathbb{R}$, for $n \in \mathbb{N}$, and a function $f \colon [0, 1] \to \mathbb{R}$.

  (a) Define what it means for f_n to *not* converge uniformly to f. (Hint: "There exists an $\epsilon > 0$ such that ...")

  (b) From the definition, prove that the sequence of functions $f_n(x) := x^n$ does not converge uniformly on the interval $[0, 1]$ to the function $f(x) := \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x = 1. \end{cases}$

Here is another couple of interesting problems.

- Spivak Chapter 24 Problem 24 (page 523).
  (Expressing a derivative f' as a pointwise limit of continuous functions.)

- Spivak Chapter 24 Problem 25 (page 523).
  (Expressing a nonintegrable function as a pointwise limit of integrable functions.)