MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 1

(1) Consider the golden ratio, $\varphi := \dfrac{1+\sqrt{5}}{2}$. Consider also $\tau := 2\pi$.

(a) Prove that $1 < \varphi < 2$.

Below, you can use the facts that $1 < \varphi < 2$ and $6 < \tau < 7$, and you may not use any other properties of φ and τ.

(b) Find a positive number E such that, for all x and y, if $|x-\varphi| < E$ and $|y-\tau| < E$, then $|xy - \tau\varphi| < 1$. Write your solution as "$E = \ldots$".

(c) Prove that the number E that you found satisfies the required condition.

(2) True or false? Discuss in one sentence.

(a) $u^2 > v^2$ implies $u > v$.

(b) $u > v$ implies $\dfrac{1}{u} < \dfrac{1}{v}$.

Here, u and v are arbitrary real numbers.

(3) For any three distinct real numbers a, b, c, we say that b is **between** a and c if either $a < b < c$ or $a > b > c$.

(a) Prove that for any three distinct real numbers exactly one of them is between the other two.

(b) Suppose that a, b, c are distinct and that b is between a and c. Find a real number t such that $0 < t < 1$ and $b = (1-t)a + tc$.

(c) Suppose that a, b, c are distinct and that t is a real number such that $0 < t < 1$ and $b = (1-t)a + tc$. Prove that b is between a and c.