MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 2

(1) Prove that, for any real number a and any two natural numbers n and m,

$$(a^n)^m = a^{nm}.$$

(Hint: see Spivak Chapter 2 Problem 23.)

(2) For any integers $0 \le k \le n$, define the binomial coefficient $\binom{n}{k}$ by

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}.$$

(a) Prove that, for all $1 \le k \le n$,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

(b) Prove that, for all $0 \le k \le n$, the binomial coefficient $\binom{n}{k}$ is an integer.

(c) Prove the binomial theorem: for any natural number n and any two real numbers a and b,

$$(a+b)^n = \sum_{j=0}^{n} \binom{n}{j} a^{n-j} b^j.$$

(Hints: see Spivak Chapter 2 Problem 3.)

(3) Let τ and e be arbitrary real numbers such that

$$6 < \tau < 7 \quad \text{and} \quad 2 < e < 3.$$

Let x and y be such that

$$|x - \tau| < \frac{1}{1000} \quad \text{and} \quad |y - e| < \frac{1}{1000}.$$

(a) Can we conclude that the distance between $x + y$ and $\tau + e$ is $< \frac{1}{100}$? Justify your answer.

(b) Can we conclude that the distance between xy and τe is $< \frac{1}{100}$? Justify your answer.