MA157 - ANALYSIS 1, 2020-2021. ASSIGNMENT 8

- (1) Using the definition of the derivative as a limit, find the derivative of the function $x \mapsto \frac{1}{\sqrt{x}}$ at a point a > 0.
- (2) (Adapted from Spivak's Chapter 9 Problem 11:)

When a chandelier falls from the ceiling, its distance from the ceiling after t seconds is a function of the form

$$s(t) = ct^2$$

for some constant c > 0 that does not depend on the mass of the chandelier.

- (a) In the 16th century, Gallileo Galilei correctly observed that the motion of a falling body is independent of the mass of the body. He also claimed that the velocity of the body is proportional to the distance that it travelled. Is this claim correct? Justify your answer.
- (b) We measure distance in meters. The accelleration of the chandelier is 9.8 meters/second. Find the value of the constant c.
- (c) The ceiling is 20 meters high.
 - (i) Around how many seconds do you have to get out of the way of the chandelier? Give an approximate answer, and justify it.
 - (ii) If you don't move out of the way of the chandelier, around how fast is the chandelier moving when it hits you? Give an approximate answer, and justify it.
- (3) (Adapted from Spivak's Chapter 10 Problem 8:)

The area between two varying concentric circles is at all times $9\pi \text{ cm}^2$. The rate of change of the area of the larger circle is $10\pi \text{ cm}^2 / \text{sec.}$ How fast is the circumference of the smaller circle changing when it has area $16\pi \text{ cm}^2$?

- (4) (Adapted from Spivak's Chapter 10 Problem 22:)
 - (a) Let $f(x) := a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$. Find a function g such that g' = f. Find another.
 - (b) Let $f(x) := \frac{b_2}{x^2} + \frac{b_3}{x^3} + \ldots + \frac{b_m}{x^m}$. Find a function g with g' = f.
 - (c) Is there a function of the form $f(x) = a_n x^n + \ldots + a_1 x + a_0 + \frac{b_1}{x} + \ldots + \frac{b_m}{x^m}$ such that $f'(x) = \frac{1}{x}$? Explain why or why not.