MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 11

Read the handout "length of a curve" on the course website.

(1) Provide the proofs for the statements in the exercise "Characterization of length" in this handout.

(2) Consider the curve $\gamma\colon [0,1] \to \mathbb{R}^2$ that is given by $\gamma(t) = (t, t\cos(\frac{\pi}{t}))$ if $t \neq 0$ and $\gamma(0) = (0,0)$.

  (a) Explain why this curve is continuous.

  (b) Consider the partition $P := \{0, \frac{1}{3}, \frac{1}{2}, 1\}$. Write a numerical expression for $\ell(\gamma, P)$. Don't simplify.

  (c) Consider the partition $P := \{0, \frac{1}{k}, \frac{1}{k-1}, \ldots, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1\}$. Show that
$$\ell(\gamma, P) \geq 1 + 1/2 + 1/3 + \ldots + 1/k.$$

(3) Prove that the curve $\gamma(t) := (t, \sqrt{t})$, for $t \in [0, 100]$, is rectifiable, and that
$$\sqrt{10,100} \leq \text{length}(\gamma) \leq 110.$$

Comment:
In Question 2(c) above, which is a computational question, you are allowed to assume properties of trigonometric functions that we did not prove. If you use such a property, please state it.