MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 12

(1) Consider the floor function $f(x) := \lfloor x \rfloor$ on the interval $[0, 5]$.
Prove that $\int_0^5 f(x)dx = 10$.

(2) Find a continuous function f on the interval $[0, 1]$, and, for each $n \in \mathbb{N}$, a partition P_n, such that the corresponding lower or upper (which?) Darboux sum is $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$.

(3) Evaluate without doing any computations; justify briefly.

(a) $\int_{-2}^{2}(2 - |x|)dx$

(b) $\int_{-2}^{2} \sqrt{4 - x^2}dx$

(c) $\int_{-5}^{5} x^3 \cos x dx$

(4) Let $f\colon [a, b] \to \mathbb{R}$ be a bounded function, and suppose that f is Darboux integrable.

(a) Prove that the function $|f|$ is Darboux integrable.

(b) Prove that $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|\, dt$.

(Based on Spivak's Ch. 13 Prob. 37.)
(Hint: show that if $m \leq y_1 \leq M$ and $m \leq y_2 \leq M$ then $||y_1| - |y_2|| \leq M - m$.)

Comment:
In Question 3 above, give a heuristic justification: you can rely on symmetry properties of the function, or on an interpretation of the integral as an area and your prior knowledge of areas of simple shapes.