MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 13

(1) (a) Consider the function $F(x) := \int_3^{e^x} \sqrt{t}dt$. Without making any computations or using any theorems, do you think that F is increasing or decreasing?

(b) Now, prove your guess. (Hint: the handout "Applications of derivatives" might be a useful refresher.)

(c) Define $G(x) := \int_1^{e^x} \sqrt{t}dt$. What are the domains of F and G? Does $F = G$? Does $F' = G'$?

(2) Let $f\colon [0,5] \to \mathbb{R}$ be a function.

(a) Write a statement that means "f is not Riemann integrable" without using the words "no" or "not".

(b) Suppose that, for every $\delta > 0$ and every $c \in \mathbb{R}$, there exists a tagged partition $(P, \{x_j\})$ with mesh $< \delta$ such that the corresponding Riemann sum satisfies $S(f, P, \{x_j\}) > c$. Prove that f is not Riemann integrable.

(3) For each of the following functions F, find an expression for the derivative $F'(x)$, without evaluating the integral.

(a) $F(x) = \int_5^{x^3} \frac{1}{1+t^4}dt$.

(b) $F(x) = \int_{-1}^{x} \left(\int_{-1}^{y} \frac{1}{2+\cos^5 t}dt \right) dy$.

(4) Let $f(x) := \int_{\pi/2}^{x} (2 + \cos(t \sin t))\, dt$.

(a) Why is f invertible? Why is the inverse function f^{-1} differentiable?

(b) Find $(f^{-1})'(0)$, without evaluating the integral.