MA157 – ANALYSIS 1, 2020–2021. ASSIGNMENT 17

(1) (From a past final exam at St.George:)

  The square with vertices $(0,5)$, $(0,7)$, $(-1,6)$, $(1,6)$ is revolved around the x-axis to make a machine belt. Find the volume of the belt.

(2) (From Apostol:)

  Consider the cone that is obtained from the region between the graph of the function $f(x) = cx$, for $0 \leq x \leq b$, and the x-axis, by rotating it about the horizontal axis. Find its volume. Show that this volume is one-third the base of the cone times the altitude of the cone.

(3) (a) Write the negation of "$\lim\limits_{n\to\infty} a_n = \ell$" without the word "limit" nor its symbol.

   (b) Rewrite the sentence "The sequence $(a_n)_{n=1}^{\infty}$ does not converge" without using the word "no" or "not". (Obtain this from the definition of "$\lim\limits_{n\to\infty} a_n = \ell$", without using any theorems.)

   (c) Prove directly that the sequence that is given by

   $$a_n := (-1)^n \left(\frac{n-1}{n}\right)$$

   does not converge. Do this directly, referring to the sentence in (b), without using any theorems.

(4) The meaning of the sentence "$\lim\limits_{n\to\infty} a_n = \ell$" begins with "For all $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that ...". Obtain a new sentence by changing the beginning to "There exists $N \in \mathbb{N}$ such that for all $\epsilon > 0$ ...". Show that this new sentence has a different meaning by giving an example of a sequence that satisfies one of these sentences but not the other.

(5) (Spivak, Chapter 22, Problem 5:)

  Prove that if $0 < a < 2$ then $a < \sqrt{2a} < 2$. Use this to prove that the following sequence converges:

  $$\sqrt{2}, \ \sqrt{2\sqrt{2}}, \ \sqrt{2\sqrt{2\sqrt{2}}}, \ \ldots$$

Find the limit of this sequence. State any theorem that you are using.