Study guide for the 2nd term test.

Coverage: everything through the end of Spivak's Chapter 11 (with the appendix), including the material of the first term test.

If you will need to use the axioms for the real numbers, then I will provide them.

Sources.

- Spivak's textbook
- The handouts, including the parts whose proofs are left to the reader as exercises.
- Notes that you took in the lectures, tutorials, office hours.
- The weightless assignments
- The for-credit assignments
- Tests and assignments from previous years

Tentative large list of relevant terms.

Older:

- "if", "only if". "for all", "exists".
- You must be able to negate statements.
- Associativity, commutativity, distributivity for addition/multiplication; additive/multiplicative inverse.
- Properties of $<$ and $\leq$: trichotomy, anti-symmetry, transitivity; relation with addition/multiplication.
- Upper/lower bound for a set. A set being bounded from above / bounded from below / bounded. Maximum/minimum of a set. Least upper bound = supremum of a set; greatest lower bound = infimum of a set.
- Least upper bound property of the real numbers. Uniqueness of least upper bound. Similar properties for greatest lower bounds.
- Archimedean property of the real numbers (for every $x \in \mathbb{R}$ there is $n \in \mathbb{N}$ such that $n > x$).
- Absolute value. Distance in $\mathbb{R}$. Distance in $\mathbb{R}^2$.
- Natural numbers = $\mathbb{N}$; integers = $\mathbb{Z}$; rational numbers = $\mathbb{Q}$.
- Recursive definition; inductive proof.
- Well ordering principle; principle of induction.
- ϵ-neighbourhood; punctured ϵ-neighbourhood; left/right ϵ-neighbourhood.
- Open/closed interval.
- Intersection/union/difference of sets. Complement of a set.
- Sum/product/composition of functions.
- Polynomial function. Degree/coefficients of a polynomial.
- Rational function.
- Dense subset of $\mathbb{R}$ (a subset A such that every open interval contains an element of A)
- Trigonometric functions; trigonometric identities.
- Domain of a function. Graph of a function.
- Even/odd function. Periodic function.
- Characteristic function of a set.
- The floor function $\lfloor x \rfloor$.
- $\lim_{x \to a} f(x) = \ell$; $\lim_{x \to \infty} f(x) = \ell$; $\lim_{x \to a} f(x) = \infty$; $\lim_{x \to a} f(x) = \infty$; Left/right limit.

- Limit of a sum, of a product, of $1/f(x)$, of $\sqrt{f(x)}$.
- Continuity of a function at a point; left/right continuity at a point.
- Continuity of a sum/product/composition.
- Pathological examples of discontinuous functions.
- Continuous and positive implies positive in a neighbourhood.

Newer:

- Continuous implies bounded in a neighbourhood.
- Continuity on an interval.
- Upper/lower bound for a function. A function being bounded from above / bounded from below / bounded. Maximum/minimum value for a function. Maximum/minimum point for a function.
- Intermediate value theorem.
- Extreme value theorem (a continuous function on a closed interval attains a maximum and a minimum).
- Monotone continuous functions.
- A function being differentiable at a point; derivative of a function at a point; tangent line for the graph of a function at a point; linear approximation for a function at a point.
- Velocity and acceleration as first and second derivatives of displacement(=location).
- Carathéodory criterion for differentiability.
- Derivatives of constant functions, linear functions, polynomials, $1/x$, $\sqrt{x}$.
- Derivative of e^x and of trigonometric functions.
- Differentiable implies continuous.
- Derivative of sum, product (Leibniz rule), composition (chain rule).
- Local maximum/minimum of a function. Critical=stationary point. Rolle's theorem.
- Mean value theorem.
- Increasing/decreasing functions.
- Vanishing derivative implies constant function; positive/negative derivative implies increasing/decreasing function.
- Second derivative at a local minimum/maximum.
- Cauchy mean value theorem; L'Hôpital's rule.
- Convex functions.