MAT157Y TERM TEST 4, APRIL 2019

Please answer all the questions; the five questions have equal weight. Clarity counts. Simplify your solutions when possible. A response "I don't know" and nothing more, to any question or section of a question, will give you 20% of the points for that question or section. You can answer the questions in any order, but please use separate pages for different questions, and mark the question number at the top corner of the page. You have 100 minutes. No aids allowed.

(1) For each of the following statements, determine if it is true or false, and add an explanatory sentence or diagram or formula. You do not need to give a complete proof.

(a) "If f is a twice differentiable function and the Taylor polynomial of order 2 for f at the point 1 is
$$5 + (x-1)^2,$$
then the Taylor polynomial of order 2 for f at the point 0 is $6 - 2x + x^2$."

(b) "If $a_n \xrightarrow[n\to\infty]{} \frac{\pi}{2}$, then $\sin(a_n) \xrightarrow[n\to\infty]{} 1$."

(c) "If f and g are functions whose their third derivatives at the point 5 exist and such that $f^{(3)}(5) \neq 0$ and $g^{(3)}(5) \neq 0$, and if $\lim_{x\to 5} \frac{f(x)-g(x)}{(x-5)^3} = 0$, then f has a local minimum at the point 5 if and only if g has a local minimum at the point 5."

(d) "The body of revolution that is obtained by rotating the square
$$\{(x,y) \mid |x-100| \leq 1 \text{ and } |y-100|^2 \leq 1\}$$
about the horizontal axis has the same volume as the body of revolution that is obtained by rotating the square
$$\{(x,y) \mid |x-99| \leq 1 \text{ and } |y-101|^2 \leq 1\}$$
about the horizontal axis."

(2) Evaluate the following limit:
$$\lim_{n\to\infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n} \right).$$
Do this by interpreting the sum as a Darboux sum or a Riemann sum for a suitable integral; specify the function, the interval, and the partition.

(3) Compute:
$$(a) \quad \int \frac{x^2}{x-1} dx \qquad\qquad (b) \quad \int e^x \sin x dx \qquad .$$

(4) Find the Taylor polynomial of order 5 of the function $f(x) = \sin x$ at the point $\frac{\pi}{2}$.

(5) Let (a_n) be a decreasing sequence of positive numbers. Prove that the sequence (a_n) converges. (Use the definitions, not a relevant theorem.)