This weightless assignment is due on Crowdmark by Wednesday, October 14, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 5, "Limits."

(a) Let f be a function, and $x_0, L \in \mathbb{R}$. Write the formal definition, and also the notation, for, "f approaches the limit L near x_0." (Remember the definition should read like a complete sentence, without any unquantified / ambiguous terms).

(b) Is the following true or false? If true, explain briefly why. If false, slightly modify the statement so that it is true.

If f approaches the limit L near x_0, then there exists an open interval I such that $x_0 \in I$ and $I \subseteq$ domain f.

Exercise 2. Read Spivak Chapter 6, "Continuity." Consider the function f defined on the interval $(0, 1)$ by

$$f(x) := \begin{cases} 0 & \text{if } x \text{ is irrational} \\ \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ in lowest terms.} \end{cases}$$

(a) At what points is this function continuous? You do not need to justify your answer.

(b) By definition $f(3/4) = 1/4 > 0$. Is there an open interval I around (i.e. containing) $1/4$ such that for every $x \in I$, $f(x) > 0$? Briefly state why or why not.

(c) Explain why your answer to (b) does not contradict Theorem 3 from Chapter 6.