This weightless assignment is due on Crowdmark by Monday, November 23, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 11, "Significance of the Derivative." Exercises 2 and 3 are also on this chapter.

(a) Suppose f is a function on $[a, b]$. What are three kinds of points where its maximum/minimum can occur? (i) (ii) (iii) . Can its max/min occur anywhere else?

(b) Draw a function whose max occurs at: only points in (i); only points in (ii); only points in (iii); at points in (i), (ii), and (iii). So you have to draw 4 graphs in total.

Exercise 2. Is each statement true or false? Justify in one sentence.

(a) If f is strictly increasing on (a, b), then $f'(x) > 0$ for all $x \in (a, b)$.

(b) Using Rolle's theorem, Spivak proves by induction that every nth degree polynomial has exactly n roots.

(c) If $f''(a) > 0$, then f has a local minimum at a.

Exercise 3. For each function, state whether we can use Theorem 7[(1)] to conclude that $f'(a)$ exists. If so, use Theorem 7 to compute $f'(a)$. If not, explain why.

(a)

$$f(x) = \begin{cases} (x-1)^2 & \text{if } x \geq 1 \\ -(x-1)^2 & \text{if } x < 1, \end{cases} \quad \text{at } a = 1$$

(b)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases} \quad \text{at } a = 0$$

(Hint: does $\lim_{x \to 0} f'(x)$ exist?)

(c)

$$f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0, \end{cases} \quad \text{at } a = 0$$

(Hint: is f continuous at 0?)

[(1)] This theorem probably seems narrow in its application, but in fact it recently came up in my own research. I was studying a 1986 result called "Joris' Theorem," and came across a proposed proof from 2003 which used only tools we will see in this course. One of them was Theorem 7, but I did not recognize it from Spivak and spent time struggling with it on my own. So, reading Spivak carefully truly is worth it! Incidentally, it turned out that 2003 proof was flawed, and other techniques are needed to prove Joris' theorem.