

This weightless assignment is due on Crowdmark by Monday, January 11, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 12, “Inverse Functions.” Consider the function defined on $(0, 2)$ by

$$f(x) := \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 2x - 1 & \text{if } x \text{ is not rational.} \end{cases}$$

- (a) Does Spivak’s version of the inverse function theorem, Theorem 5, apply to f at the point $x = 1$? Why or why not?
- (b) Is f^{-1} differentiable at 0? If so, what is $(f^{-1})'(0)$? Explain briefly, without a proof.

Exercise 2. Read Spivak Chapter 12, Appendix, “Parametric Representation of Curves.” Consider two parametrizations, for $t \in [-\pi, \pi)$

$$\begin{aligned} c_1(t) &:= (\cos t, 2 \sin t) \\ c_2(t) &:= (\cos t^3, 2 \sin t^3). \end{aligned}$$

- (a) Sketch the curve(s) parametrized by c_1 , and by c_2 . (Hint: begin by thinking about the similar parametrization $(\cos t, \sin t)$).
- (b) According to Spivak’s definition, does c_1 have a tangent line through $c_1(0)$? Does c_2 have a tangent line through $c_2(0)$? For each of these, give the equation if the tangent exists.