This weightless assignment is due on Crowdmark by Monday, January 18, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 13, "Integrals."

(a) When he proves $\int_0^b x dx = \frac{b^2}{2}$, does Spivak evaluate $L(f, P)$ and $U(f, P)$ for *any* partition P, or only some special partitions?

(b) Let $B := \{-2, 0, 1, 3\}$. Is the function $f(x) := (1 - 1_B(x))x^2 \sin\frac{1}{x}$ (taking $f(0) := 2$) integrable on $[-4, 5]$? Do not make any computations!

(c) Suppose f is integrable on $[a, b]$. Define the functions on $[a, b]$:

$$F(x) := \int_a^x f(y)dy, \quad G(z) := \int_a^z f(x)dx, \quad H(t) := \int_a^t f(V)dV.$$

Are these all the same function?

Exercise 2. Read Spivak Chapter 13, Appendix, "Riemann Sums." Set $f(x) := e^x$ on $[-1, 2]$. Let $P := \{-1, -0.5, 0.5, 1.5, 2\}$, a partition of $[-1, 2]$. Write two Riemann sums of f for P. You do not need to simplify.

[Extra: what are the sums $L(f, P)$ and $U(f, P)$? Evaluate everything with a calculator and "verify" that $L(f, P) \leq$ the Riemann sums $\leq U(f, P)$].