This weightless assignment is due on Crowdmark by Monday, January 25, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 14, "The Fundamental Theorem of Calculus."

(a) Consider the statement

Every ——————— function f on $[a, b]$ is the derivative of some function.

For the statement to be true, can we fill the blank with

   (i) Continuous?

  (ii) Lipschitz?

 (iii) Integrable?

 (iv) Bounded?

When the statement is true, give some function F such that $F' = f$. [Extra: can you find F satisfying $F\left(\frac{b-a}{2}\right) = 0$?]. When it is false, give a counter-example. Do not justify.

(b) On $[0, 2\pi]$, define $f(x) := \sin x$ and $g(x) := -\frac{1}{2}\sin\left(\frac{1}{2}x\right)$. Shade the region bounded by the graphs of f and g. Which integral(s) give its area?
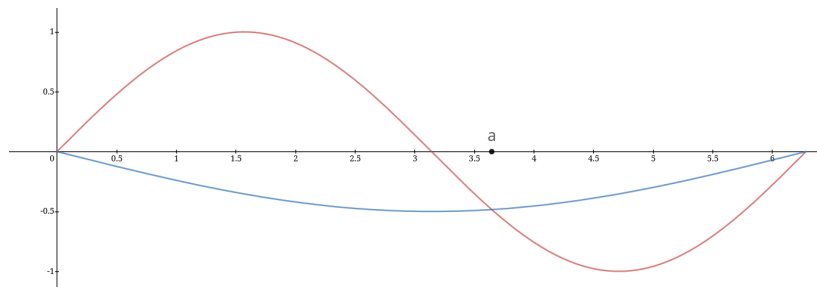
$$I_1 := \int_0^{2\pi} f - g, \quad I_2 := \int_0^{2\pi} |f - g|, \quad I_3 := \int_0^a f - g + \int_a^{2\pi} g - f, \quad I_4 := -\int_{2\pi}^0 |f - g|$$

Which one would you choose to actually compute the area explicitly?