This weightless assignment is due on Crowdmark by Monday, March 15, at 9:00pm EST. It does not count toward your course grade.

Exercise 1. Read Spivak Chapter 23, "Infinite Series."

(a) Each argument below is wrong. In each case, point out the convergence test attempted, and where the error lies.

(i) Set $a_n = 1/n^2$ and $b_n = 1/n$. Since $0 \le a_n \le b_n$ for all n, and $\sum_{n=1}^{\infty} a_n$ converges, so does $\sum_{n=1}^{\infty} b_n$.

(ii) Set $a_n = \frac{1}{n!e^n}$. Since $a_n > 0$ for all n, and

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{e(n+1)} = 0,$$

the convergence test we are using is inconclusive.

(iii) Set $a_n = \frac{\log n}{n}$, for $n \ge 4$. Define $f(x) := \frac{\log x}{x}$, on $[4, \infty)$. Since f is positive and decreasing on $[4, \infty)$, and $\int_4^{\infty} f(x)dx$ converges, so does $\sum_{n=4}^{\infty} a_n$.

(iv) Set $a_n = \frac{1}{\sqrt{n}}$ and $b_n = \frac{1}{n^2}$. Since

$$\lim_{n\to\infty} a_n/b_n = \lim_{n\to\infty} n^{3/2} = \infty \ne 0,$$

and since $\sum_{n=1}^{\infty} b_n$ converges, so does $\sum_{n=1}^{\infty} a_n$.

(b) Set $a_n = \frac{1}{\sqrt{n}}$. Justify whether the following is true or false: The infinite series

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots = \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converges, say to S. Moreover, every rearrangement of $\{(-1)^n a_n\}$ converges to the same number S.