Study guide for the midterm.

Topics:

- Euclid
- Incidence geometry
- Perspective geometry
- Neutral geometry – through Oct.18th.

You need to

- Know definitions and examples.
- Know theorems by names.
- Be able to give a simple proof and justify every step.
- Be able to explain those of Euclid's statements and proofs that appeared in the book, the problem set, the lecture, or the tutorial.

Sources:

- Class Notes
- Problem Sets
- Assigned reading
- Notes on course website.

The test.

- Excerpts from Euclid will be provided if needed.
- The incidence axioms will be provided if needed.
- Postulates of neutral geometry will be provided if needed.
- There will be at least one question that is similar or identical to a homework question.

Tentative large list of terms. For each of the following terms, you should be able to write two sentences, in which you define it, state it, explain it, or discuss it.

Euclid's geometry:

- Euclid's definition of "right angle".
- Euclid's definition of "parallel lines".
- Euclid's "postulates" versus "common notions".
- Euclid's working with geometric magnitudes rather than real numbers.
- Congruence of segments and angles.
- "All right angles are equal".
- Euclid's fifth postulate.
- "The whole is bigger than the part".
- "Collapsing compass".
- Euclid's reliance on diagrams.
- SAS; SSS; ASA; AAS.
- Base angles of an isosceles triangle.
- Angles below the base of an isosceles triangle.
- Exterior angle inequality.
- Alternate interior angle theorem and its converse (we use the textbook's convention).
- Vertical angle theorem.

- The triangle inequality.
- Pythagorean theorem.

Incidence geometry:

- collinear points.
- concurrent lines.
- Simple theorems of incidence geometry.
- Interpretation/model for a theory.
- Isomorphism of interpretations/models.
- Three-point plane.
- Four-point plane.
- Five-point plane.
- Great circles on S^2.
- Antipodes on S^2.
- S^2/antipodes.
- Cartesian plane: $\mathbb{R}^2$.
- Real projective plane: $\mathbb{RP}^2$.
- Isomorphism from $\mathbb{RP}^2$ to S^2/antipodes.
- Euclidean parallel property (=Euclidean parallel postulate).
- Hyperbolic parallel property.
- Elliptic parallel property.
- Parallel properties being independent of the incidence axioms.
- Incidence axioms being independent of each other.
- Transitivity of parallelism.
- Point at infinity.
- Projective completion.

Perspective geometry:

- Central projection to a plane in $\mathbb{R}^3$.
- Vanishing point.
- Vanishing line ("horizon").
- Geometry and algebra of lines and planes in $\mathbb{R}^3$.
- Projective completion of plane in $\mathbb{R}^3$.

Neutral geometry:

- Properties of distance that follow from the Ruler Postulate.
- "Betweenness" relation $A \star B \star C$.
- Properties of "betweenness": symmetry, "betweenness theorem for points" (distances add), trichotomy.
- Consistency of "betweenness".
- Order relations between four points on a line.
- Segments and rays.
- Convex sets.
- Extremal points; passing points.
- Endpoints and interior points of segments and rays.

(The Plane Separation Postulate and its consequences are *not* covered in the midterm.)