MAT402 Classical Geometries, Fall 2016. Problem set 2.

Read with attention Chapter 2 of John Lee's book.

You are encouraged to work in a group, but you must write your solution later, separately, on your own.

Due Friday Sept.23rd in tutorial:

- If you worked in a group, please indicate with whom you worked.
- Copy the following sentence at the top of your submission, and sign it when you're done preparing your submission: "I declare that I wrote these solutions entirely on my own."

(1) Find the flaw in the "proof" on the next page and explain it.

(2) Discuss in one paragraph: is there an "SSA" triangle congruency theorem? You may include a diagram or diagrams if you find this helpful.

(3) Prove each of the following theorems of Incidence Geometry directly from the axioms of Incidence Geometry.
  - (a) Given any line, there exists a point that does not lie on it.
  - (b) If A, B, C are distinct non-collinear points, then $\overleftrightarrow{AB} \neq \overleftrightarrow{AC}$.
  - (c) Given any two distinct non-parallel lines, there exists a unique point that lies on both of them.
  - (d) There exist three non-concurrent lines.

(4) Use interpretations/models to show that each of the incidence axioms is independent of the other three.

(5) Find a model of incidence geometry that has exactly four points but that is not isomorphic to the four-point plane. Describe this model, explain why it is a model of incidence geometry, and explain why it is not isomorphic to the four-point plane.

A flawed "proof":

EXAMPLE 3. *If two parallel straight lines are intersected by a third line, the sum of the interior angles lying on the same side of the third line is equal to* 180° (*proof not based on the parallel postulate*).

Say $AB \parallel CD$, and let the line EF intersect these two lines (Fig. 3). The interior angles are designated by numbers in the drawing.

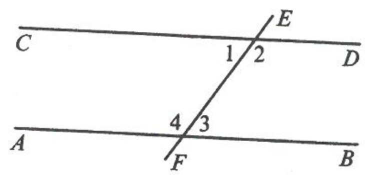

Fig. 3

Three cases are possible:[1]

1. The sum of the interior angles lying on the same side of the line EF is $> 180°$.
2. The sum of the interior angles on the same side of the line EF is $< 180°$.
3. The sum of the interior angles on the same side of the line EF is $= 180°$.

In the first case we have

$$\angle 1 + \angle 4 > 180°, \quad \angle 2 + \angle 3 > 180°;$$

therefore,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 > 360°.$$

But the sum of the four interior angles is equal to two straight angles, that is, 360°. This contradiction shows that the first assumption must be discarded. For the same reason we must also abandon the second assumption, as it would lead to the conclusion that the sum of the four interior angles is less than 360°. The third assumption is the only possible one (it does not lead to a contradiction); this proves the theorem.

[1] Here and subsequently, when talking about possible assumptions or possible cases, we do not by any means assert that they are all actually possible under the conditions of the given example. On the contrary, time and again it will happen that what is at first assumed to be a possible case later turns out to be spurious—contrary to the conditions or to what is taken as established; this often happens in indirect proofs. Thus, we are talking throughout about so-called "*a priori* possibilities," that is, about possibilities which present themselves beforehand, prior to taking into account the other conditions of the problem.

From:
"Mistakes in Geometric Proofs",
by Ya. S. Dubnov