MAT402 Classical Geometries, Fall 2016. Problem set 3.

You are encouraged to work in a group, but you must write your solution later, separately, on your own.

Due Friday Sept.30th in tutorial:

- If you worked in a group, please indicate with whom you worked.
- Copy the following sentence at the top of your submission, and sign it when you're done preparing your submission: "I declare that I wrote these solutions entirely on my own."

(1) Find the flaw in the "proof" on the next page and explain it.

(2) Solve the Exercise 2I from Page 51 of John Lee's textbook. (A finite model with interesting parallelism behaviour.)

(3) The **rational plane** is the following interpretation of incidence geometry. A "point" is interpreted to be an ordered pair (x, y) of rational numbers $x, y \in \mathbb{Q}$. A "line" is interpreted to be a set of points of one of the following forms.

$$\ell = \{(x, y) \in \mathbb{Q}^2 \mid y = mx + b\} \qquad \text{for some } m, b \in \mathbb{Q} \ ,$$

or

$$\ell = \{(x, y) \in \mathbb{Q}^2 \mid x = c\} \qquad \text{for some } c \in \mathbb{Q} \ .$$

We call them **rational points** and **rational lines**.

The rational plane is a model for incidence geometry. In this exercise you are asked to check two of the four incidence axioms for this model.

(a) Let (x_1, y_1) and (x_2, y_2) be two distinct rational points. Prove that there exists a rational line that contains both of them.

(b) Let ℓ be a rational line. Prove that it contains at least two distinct rational points.

(4) Prove, from the axioms of incidence geometry:

(a) There exist at least three distinct lines.

(b) For every line ℓ and every two distinct points A, B that lie on ℓ,

$$\ell = \overleftrightarrow{AB}.$$

(5) Let P be a model for incidence geometry that satisfies the Euclidean parallel property. Let $\widehat{P}$ be its projective completion. Show that every line in $\widehat{P}$ contains at least three points of $\widehat{P}$.

A flawed "proof":

EXAMPLE 4. *The sum of the angles of a triangle is equal to* 180° (*proof not based on the parallel postulate*).

Divide the arbitrary triangle ABC into two triangles by means of a line segment drawn from the vertex, and denote the angles by numbers as in Fig. 4. Let x be the sum of the angles of a triangle, unknown as yet; then

$$\angle 1 + \angle 2 + \angle 6 = x,$$
$$\angle 3 + \angle 4 + \angle 5 = x.$$

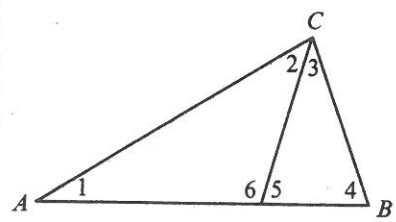

Fig. 4

Adding these two equalities, we obtain

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 2x.$$

But the sum $\angle 1 + \angle 2 + \angle 3 + \angle 4$ is the sum of the angles of the triangle ABC, that is, it is also x; and the angles 5 and 6, being adjacent angles, have a sum equal to 180°. Thus, for finding x we have the equation $x + 180° = 2x$, from which it follows that $x = 180°$.

From:
"Mistakes in Geometric Proofs,"
by Ya. S. Dubnov