MAT402 Classical Geometries, Fall 2016. Problem set 4.

Read your class notes about projective spaces, lines and planes in $\mathbb{R}^3$, and perspective geometry. Read Appendices E and F of John Lee's textbook. You are encouraged to work in a group, but you must write your solution later, separately, on your own. Due Friday Oct.7th in tutorial:

- If you worked in a group, please indicate with whom you worked.
- Copy the following sentence, and sign it when you're done preparing your submission: "I declare that I wrote these solutions entirely on my own."

(1) Solve Exercise EC parts (a), (c), (e) on Page 403 of the textbook.

(2) Consider the real projective plane $\mathbb{RP}^2$. Recall that, using homogeneous coordinates, a point in $\mathbb{RP}^2$ can be written in the form $[a, b, c]$ with $a, b, c \in \mathbb{R}$ not all zero. Recall that a line in $\mathbb{RP}^2$, being a two dimensional subspace of $\mathbb{R}^3$, can be given by a linear equation $Ax_1 + Bx_2 + Cx_3 = 0$ with $A, B, C \in \mathbb{R}$ not all zero, where x_1, x_2, x_3 are the coordinates on $\mathbb{R}^3$.

(a) Find the line in $\mathbb{RP}^2$ that contains the points $[1, 1, 1]$ and $[0, 1, 3]$. (Describe it by an equation.)

(b) Find the point in $\mathbb{RP}^2$ where the lines $x + y + z = 0$ and $y + 3z = 0$ meet.

(3) Consider the projective completion of the Cartesian plane $\mathbb{R}^2$. We use the following shorthand notation: for a Cartesian line ℓ that is given by the equation $ax + by = c$, we denote the corresponding point at infinity, $[\ell]$, by $[ax + by = c]$. Also, we denote the line at infinity by ℓ_∞.

(a) Find the line in the projective completion that passes through both the Cartesian point $(2, 1)$ and the point at infinity $[5x + 10y = 1]$.

(b) Find the point in the projective completion that lies on both the line at infinity ℓ_∞ and the Cartesian line that is given by $x - y = 1$.

(c) Find the point in the projective completion that lies on both the Cartesian line that is given by $-2x + y = 3$ and the Cartesian line that is given by $4x - 2y = 5$.

(4) Consider $\mathbb{R}^3$ with the coordinates x_1, x_2, x_3. Let P be the plane given by $x_2 = 1$. Consider central projection to P with respect to the origin.

(a) Find the plane P_0 through the origin that is parallel to P.

(b) Find the image of a point $(a, b, c) \in \mathbb{R}^3$ under the central projection to P. When is this image not well defined?

(c) Consider the line $\ell = \{(1, 2, 3) + t(0, 0, 1) \mid t \in \mathbb{R}\}$. Find its image under the central projection to P. Write this image in the form $\{q + tv \mid t \in \mathbb{R}\}$ for some $q, v \in \mathbb{R}^3$.