MAT402 Classical Geometries, Fall 2016. Problem set 5.

Read Chapter 3 of John Lee's textbook.

This problem set is due after reading week, on Tuesday Oct.18th in class. You are encouraged to work in a group, but you must write your solution later, separately, on your own.

- If you worked in a group, please indicate with whom you worked.
- Copy the following sentence, and sign it when you're done preparing your submission: "I declare that I wrote these solutions entirely on my own."

(1) The central projection to the plane $P = \{x_2 = 1\}$, with respect to the origin, sends the line $S = \{(1, 0, 1) + t(1, 2, 0) \mid t \in \mathbb{R}\}$ to a line S' in P. Describe S' in the form $\{q + tv \mid t \in \mathbb{R}\}$, and find the vanishing point in S'.

(2) Consider an affine line, given by $S = \{q + tv \mid t \in \mathbb{R}\}$ where $q, v \in \mathbb{R}^3$ and $v \neq 0$. The corresponding one dimensional subspace of "directions along S" is given by $S_0 = \{tv \mid t \in \mathbb{R}\}$. Assume that the vectors q and v are linearly independent. Let $E = \operatorname{span}\{q, v\}$.
  (a) Show that the line S does not pass through the origin of $\mathbb{R}^3$.
  (b) Show that the plane E contains the origin and contains S.
  (c) Show that, for any $x \in S$, the "line of sight" $\{sx \mid s \in \mathbb{R}\}$ is contained in E.
  (d) Show that, for every one dimensional subspace L of E, if $L \neq S_0$ then there exists an $x \in S$ such that L coincides with the "line of sight" $L = \{sx \mid s \in \mathbb{R}\}$.

(3) Prove the "Ruler flipping lemma" of Neutral geometry: if $f\colon \ell \to \mathbb{R}$ is a coordinate function for line ℓ, then the function $f_2\colon \ell \to \mathbb{R}$ defined by $f_2(x) = -f(x)$ is also a coordinate function for ℓ.

This is Lemma 3.4 of the textbook. In proving it, you may assume Postulates 1–5 of Neutral geometry: the Set postulate, the Existence postulate, the Unique line postulate, the Distance postulate, and the Ruler postulate.

(4) Prove the "Segment construction theorem" of Neutral geometry: Given a ray $\overrightarrow{AB}$ and a positive real number r, there exists a unique point C on $\overrightarrow{AB}$ such that $AC = r$.

The textbook version is Theorem 3.35. In proving it, you may use any Postulates/Definitions/Theorems etc that precede this theorem in Chapter 3.

(5) Prove that, in Neutral geometry, every point lies on infinitely many distinct lines. You may use results from Chapter 3 of the book. (Hint: given a point A, take a line ℓ that does not contain A, and show that if B, C are distinct points on ℓ then the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are distinct.)