MAT402 Classical Geometries, Fall 2016. Problem set 9

Read Chapters 6 and 7 of John Lee's textbook.

This problem set is due in class on Friday Nov.25th. You are encouraged to work in a group, but you must write your solution later, separately, on your own.

- If you worked in a group, please indicate with whom you worked.
- Copy the following sentence, and sign it when you're done preparing your submission: "I declare that I wrote these solutions entirely on my own."

As usual, when proving a theorem in Neutral geometry that appears in our textbook, you may use parts of Neutral geometry that appear earlier in the textbook: postulates, theorems (or lemmas/propositions/corollaries etc), primitive notions, and definitions.

(1) Let $\overrightarrow{r}$ be a ray starting at point O. Let ℓ be a line through O. Prove:
  (i) If $\ell = \overleftrightarrow{r}$, then $\ell \cap \overrightarrow{r} = \overrightarrow{r}$.
  (ii) If $\ell \neq \overleftrightarrow{r}$, then $\ell \cap \overrightarrow{r} = \{O\}$.

  Let $\angle ab$ be a proper angle with vertex O. Let ℓ be a line through O. Prove:
  (iii) If $\ell = \overleftrightarrow{a}$ then $\ell \cap \angle ab = \overrightarrow{a}$. If $\ell = \overleftrightarrow{b}$ then $\ell \cap \angle ab = \overrightarrow{b}$.
  (v) If $\ell \neq \overleftrightarrow{a}$ and $\ell \neq \overleftrightarrow{b}$ then $\ell \cap \angle ab = \{O\}$.

  Let $\angle ab$ and $\angle cd$ be angles. Suppose that $\angle ab = \angle cd$. Prove:
  (vi) $\angle ab$ is a proper angle if and only if $\angle cd$ is a proper angle.
  (vii) Suppose that $\angle ab$ and $\angle cd$ are proper angles. Then $\overrightarrow{c} = \overrightarrow{a}$ or $\overrightarrow{c} = \overrightarrow{b}$.
  (viii) Suppose that $\angle ab$ and $\angle cd$ are proper angles. Then $\overrightarrow{c} = \overrightarrow{a}$ and $\overrightarrow{d} = \overrightarrow{b}$ or $\overrightarrow{c} = \overrightarrow{b}$ and $\overrightarrow{d} = \overrightarrow{a}$.

(2) Prove the Isosceles Triangle Altitude Theorem: In an isosceles triangle, the altitude to the base coincides with the median to the base and with the bisector of the angle opposite the base (Theorem 7.6 of the textbook).

(3) Prove that every segment has a unique perpendicular bisector (Theorem 7.7 of the textbook).

(4) Prove Lemma 7.12 of the textbook ("Properties of Closest Points").

(5) Prove Theorem 7.13 of the textbook ("Closest Point on a Line").

Additional questions, for you to solve but not to hand in:

- Prove the Corresponding Angles Theorem (Theorem 7.20 of the textbook).
- Prove the Consecutive Interior Angles theorem (Theorem 7.21 of the textbook).
- Show that "Euclid's Segment Cutoff Theorem" is false in the rational plane by constructing explicit segments $\overline{AB}$ and $\overline{CD}$ with rational endpoints such that $AB > CD$ but such that there is no rational point $E \in \overline{AB}$ with $\overline{AE} \cong \overline{CD}$. ("Rational point" is a pair (x, y) with x, y rational numbers. See Example 6.20 in the textbook.)
- Show that the taxicab geometry that is described in Chapter 6 of the textbook satisfies the ruler postulate. See the hints provided in Exercise 6C on page 140 of the textbook.